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ON THE TRANSLATIONALLY-INVARIANT SOLUTIONS OF THE MEMBRANE SHAPE EQUATION

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Abstract. The membrane shape equation derived by Helfrich and Ou-Yang describes the equilibrium shapes of biomembranes, built by bilayers of amphiphilic molecules, in terms of the mean and Gaussian curvatures of their middle-surfaces. Here, we present a new class of translationally-invariant solutions to this equation in terms of the elliptic functions which completes the solutions found earlier. In this way, all translationally-invariant solutions to the membrane shape equation are determined. Special attention is paid to those translationally-invariant solutions of the membrane shape equation which determine closed cylindrical (tube-like) surfaces (membrane shapes). Several examples of such surfaces are presented.

1. Introduction

Within the framework of the Helfrich spontaneous curvature model [3], the equilibrium shapes of a biomembrane, assumed as a bilayer of amphiphilic molecules (phospholipids, for instance), are described in terms of the mean H and Gaussian K curvatures of its middle-surface $\mathcal{S}$ by the **membrane shape equation** [7, 8]

$$2k_c\Delta H + k_c\left(2H + \mathbb{h}\right)\left(2H^2 - \mathbb{h}H - 2K\right) - 2\lambda H + p = 0 \qquad (1)$$

where k_c, $\mathbb{h}$ and λ are real constants representing the bending rigidity, spontaneous curvature and tensile stress of the membrane, respectively, while p is the osmotic pressure difference between the outer and inner media assumed to be a real constant too. Here, Δ is the **Laplace–Beltrami operator** on the surface $\mathcal{S}$.