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# COMPATIBLE POISSON TENSORS RELATED TO BUNDLES OF LIE ALGEBRAS

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**Abstract.** We consider some recent results about the Poisson structures, arising on the co-algebra of a given Lie algebra when we have on it a structure of a bundle of Lie algebras. These tensors have applications in the study of the Hamiltonian structures of various integrable nonlinear models, among them the $\mathrm{O}(3)$-chiral fields system and Landau–Lifshitz equation.

## 1. Introduction

Suppose that $\mathrm{Mat}(n, \mathbb{K}) \equiv \mathrm{End}(\mathbb{K}^n)$ is the linear space of all $n \times n$ matrices over the field $\mathbb{K}$, which willl be either $\mathbb{R}$ or $\mathbb{C}$ and will be specified explicitly only if it is necessary. $\mathrm{Mat}(n)$ possesses a natural structure of associative algebra and as a consequence – a structure of a Lie algebra defined by the commutator $[X, Y] = XY - YX$, denoted by $\mathfrak{gl}(n)$. However, the structure of the associative algebra over $\mathrm{Mat}(n)$ is not unique, indeed, if we fix $J \in \mathrm{Mat}(n)$, then we can define the product $(X \circ Y)_J = XJY$ and with respect to it $\mathrm{Mat}(n)$ is again an associative algebra. This induces a new Lie algebra structure, defined by the bracket

$$[X, Y]_J = XJY - YJX. \tag{1}$$

Thus we obtain a family of Lie brackets, labelled by the element $J$. It is readily seen that we actually have a linear space of Lie brackets, since the sum of two such brackets is also a Lie bracket of the same type. The above construction can be applied even if $X, Y, J$ are not $n \times n$ matrices, since (1) makes sense when $X, Y \in \mathrm{Mat}(n, m)$ – the linear space of $n \times m$ matrices and $J \in \mathrm{Mat}(m, n)$ – the linear space of $n \times m$ matrices. According to the accepted terminology, (1) defines a linear bundle of Lie algebras. Another example is obtained if we take $X$,