Geometry,
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THE RELATIVISTIC HYPERBOLIC PARALLELOGRAM LAW

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Abstract. A gyrovector is a hyperbolic vector. Gyrovectors are equivalence classes of directed gyrosegments that add according to the gyroparallelogram law just as vectors are equivalence classes of directed segments that add according to the parallelogram law. In the "gyrolanguage" of this paper one attaches the prefix "gyro" to a classical term to mean the analogous term in hyperbolic geometry. The prefix stems from Thomas gyration, which is the mathematical abstraction of the relativistic effect known as Thomas precession. Gyrolanguage turns out to be the language one needs to articulate novel analogies that the classical and the modern in this paper share. The aim of this article is to employ recent developments in analytic hyperbolic geometry for the presentation of the relativistic hyperbolic parallelogram law, and the relativistic particle aberration.

1. Introduction

Einstein noted in 1905 that

> "Das Gesetz vom Parallelogramm der Geschwindigkeiten gilt also
> nach unserer Theorie nur in erster Annäherung."

> A. Einstein [1]

[Thus the law of velocity parallelogram is valid according to our theory only to a first approximation.] The important "velocity parallelogram" notion that appears in Einstein's 1905 original paper [1] as "Parallelogramm der Geschwindigkeiten" does not appear in its English translation [2]. It can be found, however, in other English translations as, for instance, the translation by H. Lorentz, H. Weyl and H. Minkowski [6, pp. 37–65; p. 50].

About a century later the geometry underlying Einstein's observation on the approximate validity of the velocity parallelogram was uncovered in [18, 23].