Fifth International Conference on
Geometry, Integrability and Quantization
June 5–12, 2003, Varna, Bulgaria
Ivaïlo M. Mladenov and Allen C. Hirshfeld, Editors
SOFTEX, Sofia 2004, pp 169–177

CURVATURE PROPERTIES OF SOME THREE-DIMENSIONAL ALMOST CONTACT MANIFOLDS WITH B-METRIC II

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Abstract. The curvature tensor on a 3-dimensional almost contact manifold with B-metric belonging to two main classes is studied. These classes are the rest of the main classes which were not considered in the first part of this work. The dimension 3 is the lowest possible dimension for the almost contact manifolds with B-metric. The corresponding curvatures are found and the respective geometric characteristics of the considered manifolds are given.

1. Preliminaries

Let $(M^{2n+1}, \varphi, \xi, \eta, g)$ be a $(2n+1)$-dimensional almost contact manifold with B-metric, i.e. (φ, ξ, η) is an almost contact structure and g is a metric on M such that:

$$\varphi^2 = -\mathrm{id} + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad g(\varphi X, \varphi Y) = -g(X, Y) + \eta(X)\eta(Y)$$

where $X, Y \in \mathcal{X}M$.

Both metrics g and its associated $\tilde{g}(X, Y) = g^*(X, Y) + \eta(X)\eta(Y)$ are indefinite metrics of signature $(n, n+1)$ [1], where it is denoted $g^*(X, Y) = g(X, \varphi Y)$.

Further, X, Y, Z, W will stand for arbitrary differentiable vector fields on M (i.e. the elements of $\mathcal{X}M$) and x, y, z, w are arbitrary vectors in the tangential space T_pM, $p \in M$.

Let $(V^{2n+1}, \varphi, \xi, \eta, g)$ be a $(2n+1)$-dimensional vector space with almost contact structure (φ, ξ, η) and B-metric g. It is well known the orthogonal decomposition $V = hV \oplus vV$ of $(V^{2n+1}, \varphi, \xi, \eta, g)$, where $hV = \{x \in V;\ x = hx = -\varphi^2 x\}$, $vV = \{x \in V;\ x = vx = \eta(x)\xi\}$. Denoting the restrictions of g and φ on hV by the same letters, we obtain the $2n$-dimensional almost complex vector space