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ON GENERALIZED FOURIER TRANSFORM FOR KAUP-KUPERSHMIDT TYPE EQUATIONS*

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Abstract. We develop the Fourier transform interpretation of the inverse scattering method for nonlinear integrable evolution equations associated with a $\mathbb{Z}_3$ reduced Zakharov-Shabat system for the Lie algebra $\mathfrak{sl}(3,\mathbb{C})$.
A simple representative of this integrable hierarchy is the well-known Kaup-Kupershmidt equation. Our results admit a natural extention for nonlinear equations connected to a deeply reduced Zakharov-Shabat system related to an arbitrary simple Lie algebra.

1. Introduction

The Kaup-Kupershmidt equation (KKE) is a $1+1$ nonlinear evolution equation

$$\partial_t f = \partial^5_{x^5} f + 10 f \partial^3_{x^3} f + 25 \partial_x f \partial^2_{x^2} f + 20 f^2 \partial_x f \tag{1}$$

where $f \in C^\infty(\mathbb{R}^2)$. It is integrable by means of the inverse scattering method: it is related to a third order spectral problem [11]

$$(\partial^3_{x^3} + 2f\partial_x + \partial_x f)y = \lambda^3 y$$

for some smooth function $y(x,t,\lambda)$. It can be easily transformed into a first order one [5] but related to the algebra $\mathfrak{sl}(3,\mathbb{C})$

$$(\mathrm{i}\partial_x + q - \lambda J)\psi = 0, \qquad q = \begin{pmatrix} u & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -u \end{pmatrix}, \quad J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

with some additional symmetries imposed and for an appropriately chosen new dependent variable u. Thus a system of the **Caudrey-Beals-Coifman** (CBC) type occurs. This is a typical situation when transforming a scalar differential operator

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