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A C-SPECTRAL SEQUENCE ASSOCIATED WITH FREE BOUNDARY VARIATIONAL PROBLEMS

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Abstract. The $\mathcal{C}$–spectral sequence is a cohomological theory naturally associated with a space of infinite jets, which allows to write down many concepts of the variational calculus by using the same logic of the standard differential calculus. In this paper we use such a language (called Secondary Calculus by A. Vinogradov) to describe a delicate aspect of the variational calculus: the appearance of some "natural" boundary conditions in the context of variational problems with free boundary (e.g., transversality conditions). We discover that the Euler–Lagrange operator is actually a graded operator, producing simultaneously the standard Euler–Lagrange equations and these new boundary conditions as different homogeneous components of an unique object. Simple applicative examples will be presented.

1. Introduction

When a system of nonlinear PDEs is formalized as a natural geometrical object, one can use the common tools of differential calculus (e.g., locality, differential cohomology, symmetries, etc.) to reveal some aspects of the equations, which could be hardly accessed by just using analytic techniques (the first steps in this direction were moved by Dedecker 1978, Gel'fand and Dikii 1975, Hordenski 1974, Olver and Shakiban 1978, Tulczyjew 1975, 1977, 1980, and Vinogradov 1977, 1978).

The present work was sustained by the belief that the right geometrical portrait of a system of nonlinear PDEs is the so–called **diffiety** (see [3, 6] and references therein) and that the analitic machinery which is commonly used to look for solutions of PDEs just draws attention away from the more conceptual, and hence more interesting, aspects of the problem.