

## QUATERNIONS AND ROTATION SEQUENCES

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**Abstract.** In this paper we introduce and define the quaternion; we give a brief introduction to its properties and algebra, and we show, what appears to be, its primary application — the quaternion rotation operator. The quaternion rotation operator competes with the conventional matrix rotation operator in a variety of rotation sequences.

### 1. Introduction

The 1950's post World War II period was a time in world history when large nations were again driven by Minds of Fear — fear of each other. The development of many new technologies continued to flourish, perhaps because of this fear. In these post-war years I was involved in the aerospace industry. On various occasions, I would meet with several people, each of whom represented one of several companies. These companies together formed a Consortium with a common goal — that of designing an anti-Inter Continental Ballistic Missile (anti-ICBM).

My interest at that time (for the Consortium) was Inertial Guidance. The proposed anti-ICBM Guidance strategies suggested by members of the Consortium often encountered orientations which approached gimbal-lock and therefore it would introduce its associated errors. At one point someone asked whether *quaternions* might offer an alternative computational approach. I didn't know — in fact, that was my first encounter with the term *quaternion*. That was quite long time ago, but it was the start of my personal foray into these matters.

In this paper we introduce and define the quaternion, give a brief introduction to its properties and its algebra. We then illustrate what is perhaps its primary application in a quaternion rotation operator. And, finally, we use these quaternion operators in a variety of rotation sequence applications.