2007 Commentary on *J-holomorphic curves and Quantum Cohomology*

We have decided to make our first set of lecture notes on J-holomorphic curves available on the Web since, despite having a few mistakes and many omissions, it is still a readable introduction to the field. It was originally published by the AMS in 1994. It is now out of print, replaced by the enlarged version *J-holomorphic curves and Symplectic Topology* (AMS 2004) that we will refer to as JHOL.

Main mistakes and omissions

1. **The proof the Calderon–Zygmund inequality** in Appendix B.2 was simply wrong. A correct proof is given in JHOL Appendix B.

2. **The proof of Gromov compactness** was short, but somewhat careless. Claim (ii) on p 56 in the proof of the Gromov compactness theorem 4.4.3 is wrong; the limiting curve v might be constant. We used the claim that v is nonconstant to show that the limiting process converges after finitely many steps.

To correct this one needs to take more care in the rescaling argument, namely the center of the rescaling must be chosen at a point where the function $|du^\nu|^2$ attains a local maximum (Step 1 in the proof of Proposition 4.7.1, p. 101 in JHOL). In this way one obtains *two* additional bubbling points at the center and on the boundary of the unit disc for the rescaled sequence, when the limit is constant (see (iv) in Proposition 4.7.1 and Step 4 in the proof, p. 104 in JHOL).

The refined rescaling argument gives as a limit a *stable map*, a crucial missing ingredient in the old version. One then uses JHOL Exercise 5.1.2 to establish a bound on the number of components of the limiting stable map.

3. **An omission in the Gluing proof in Appendix A.** The proof of the existence of the gluing map f_R of Theorem A.5.2 is basically all right, but somewhat sketchy. In particular we did not prove in detail that it is a local diffeomorphism. We claim in the middle of p. 175 that this follows from the uniqueness result in Proposition 3.3.5. While this is true, several intermediate steps are needed for a complete proof. Full details are given in JHOL Chapter 10.

4. **An omission in the transversality argument.** In Chapter 2 we established basic results on the structure of simple J-holomorphic curves only in the case of C^∞-smooth J. However, in the application in Chapter 3 we need these results for J of class C^ℓ with $\ell < \infty$. This was somewhat concealed: the statement of Proposition 3.4.1 does not make clear that the elements in the universal moduli space $\mathcal{M}^\ell(A, J)$ are assumed to be *simple* although we used that assumption in the proof.

Additions in JHOL

Many of the discussions in this book are quite sketchy. In JHOL they are all fleshed out, and there are many more examples given. Here are other main additions:

• The existence of Gromov–Witten invariants is established in more generality (though the argument still needs some version of the "semipositive" hypothesis). Chapter 7 discusses the axioms they satisfy and also has a completely self-contained section on the very special and important example of rational curves in projective space;

• a treatment of genus zero stable curves and maps (in Appendix D and Chapter 5);

• a chapter on geometric applications of genus zero J-holomorphic curves without using gluing (Chapter 9);

• a discussion of Hamiltonian Floer homology and applications such as spectral invariants, explaining the set up but without doing the basic analytic proofs (Chapter 12);

• much more detailed appendices on the analysis. Also an appendix giving full details of a new proof (due to Lazzarini) on positivity of intersections.

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