

compactification, the normal bundle is anti-ample; which explains in another way why we have the nonpositive powers of this invertible sheaf.)

Since  $V$  is of Hodge type  $\{(-1,0),(0,-1)\}$ , we can also consider the mixed Shimura data  $V \times (P, X)$ . The resulting family of abelian varieties inherits multiplication by  $E$ , which leads to a modular interpretation of  $M^{Kf}(P, X)$ . A suitable extension of this family in terms of toroidal compactification can, as in 10.17-10.22, be described as degeneration of such abelian varieties. All these structures can, of course, be described in terms of the morphisms in 12.4.

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List of frequently used symbols

$A, A_f, A_K, A_{K,f}$	adele ring	0.2
$\text{Ad}_G, \text{ad}_G$	adjoint operation	0.3
$A[d]$	group of $d$ -division points	10.1
$B(X^0), B(X^0, P_1)$	set of commutators	8.11
$\beta(P_1, X_1, P'_1, X'_1, p_f), \bar{\beta}(P_1, X_1, P'_1, X'_1, p_f)$	certain maps in $\mathcal{U}, \bar{\mathcal{U}}$	6.11, 6.15
$\mathbb{C}$	set of complex numbers	0.2
$\mathcal{C}(P, X)$	conical complex	4.24
$C(X^0, P_1)$	open cone associated to boundary component	4.15
$C^*(X^0, P_1)$	union of all boundary components of $C(X^0, P_1)$	4.22
$(\text{CSp}_{2g}, \mathcal{H}_{2g})$	Shimura data for the symplectic group	2.7
$\Delta_1$	normalizer of a boundary stratum in the Baily-Borel compactification	6.3, 6.18, 7.3
$e_\sigma$	splitting associated to a cone $\sigma$	5.11, 8.5
$\exp L$	subgroup associated to sub-Lie algebra	0.3
$E(P, X)$	reflex field	11.1
$e(X)$	canonical pairing $H(X) \times H(X) \rightarrow \mathbb{G}_m$	10.2
$\mathcal{E}(X^0, P_1)$	set of splittings	9.2
$\text{FPM}_{\mathbb{C}}$	Hodge filtration	1.1
$G$	semisimple part $P/W$	2.1
$G^{\text{ad}}, G^{\text{der}}$	adjoint, derived group	0.3
$(\mathbb{G}_{m,0}, \mathcal{H}_0), (\mathbb{G}_{m,0}, h(\mathcal{H}_0))$	standard Shimura data	2.8
$h$	homomorphism $\mathcal{S}_{\mathbb{C}} \rightarrow P_{\mathbb{C}}$	1.4
$h$	equivariant map $X \rightarrow \text{Hom}(\mathcal{S}_{\mathbb{C}}, P_{\mathbb{C}})$	2.1
$H_0, h_0, h_*$	reference group and homomorphisms	4.3
$H(X)$	group of translations normalizing $X$	10.1
$\mathcal{H}_W$	conjugacy class of $h: \mathcal{S}_{\mathbb{C}} \rightarrow P_{\mathbb{C}}$	1.6

$im$	imaginary part	4.14
$int_G$	interior automorphism	0.3
$int(p)$	interior automorphism of $(P, X)$	3.5
$K_f(d), K_f^P(d), K_f^U(d), K_f^W(d)$	special open compact subgroups	10.7, 10.15
$\mathcal{L}_P$	canonical invertible sheaf on $M^{K_f}(P, X)^*(\mathbb{C})$	8.1
$\mathcal{L}_X$	invertible sheaf occurring in $q$ -expansion	12.18
$\lambda$	isomorphism $Z \rightarrow Z(1)$	2.8, 3.16, 8.5
$M^{p,q}$	Hodge decomposition	1.1
$M(n)$	Tate twist	1.11
$M^{K_f}(P, X)(\mathbb{C}), M_C^{K_f}(P, X), M^{K_f}(P, X)$	(mixed) Shimura variety	3.1, 9.25, 11.5
$M^{K_f}(P, X)^*(\mathbb{C}), M_C^{K_f}(P, X)^*, M^{K_f}(P, X)^*$	Baily-Borel compactification	6.2, 9.25, 12.3
$M^{K_f}(P, X)(\mathbb{C}), M_C^{K_f}(P, X), M^{K_f}(P, X)$	toroidal compactification	6.24, 9.25, 9.34, 12.4-5
$M^{K_f}(P, X)^*(\mathbb{C})$	union of $M^{K_f}(P, X)(\mathbb{C})$ with all boundary strata of codimension 1	8.2
$M(d), M_V(d), M_W(d), M^0(d), M_U^0(d)$	special mixed Shimura varieties	10.7, 10.15
$\mathbb{A}_d$	moduli scheme of abelian varieties	10.6, 11.16
$M_d^0, X_d^0$	"moduli scheme" of roots of unity	10.15
$\mu$	canonical cocharacter of $S$	1.3
$ord_T$	homomorphism $T(\mathbb{C}) \rightarrow Y_*(T)_{\mathbb{R}}$	5.8
$\omega_{X/S}^T$	sheaf of invariant differentials	5.26
$\omega\{dlog\}, \omega_{X_2}\{dlog\}, \omega_{X_2/S}\{dlog\}$	sheaf of differentials with at most logarithmic poles along the boundary	5.26, 8.1
$(P, X), (P, X, h)$	(mixed) Shimura data	2.1
$(P, X)/P_0$	quotient mixed Shimura data	2.9
$(P_1, X_1) \times_{(P, X)} (P_2, X_2)$	fibre product	2.20

$(P_0, X_0)$	unipotent extension of $(\mathbb{G}_m, \mathbb{Q}, \mathcal{H}_0)$	2.24
$(P_{2g}, X_{2g})$	unipotent extension of $(\text{CSp}_{2g}, \mathcal{H}_{2g})$	2.25
$(P_{[\sigma]}, X_{[\sigma]})$	mixed Shimura data associated to $[\sigma]$	7.1
$\pi$	the positive real number	0.2
$\pi, \pi'$	projections $P \rightarrow G = P/W, P \rightarrow P/U$	2.1
$\pi_0(X)$	set of connected components	0.1
$\pi_\sigma$	canonical projection $T_\sigma \rightarrow \tilde{T}_\sigma$	5.2
$\pi_{[\sigma]}$	projection $(P, X) \rightarrow (P_{[\sigma]}, X_{[\sigma]})$	7.1
$[\pi]^*$	projection to Baily-Borel compactification	6.24, 9.25, 12.4
$Q$	admissible $Q$ -parabolic subgroup	4.5
$\mathcal{R}_{L/K} X$	Weil restriction	0.2
$\mathbb{R}$	set of real numbers	0.2
$S, S^1$	Deligne torus	1.3
$\delta, \delta(X^0, P_1, P_f)$	admissible (partial) cone decomposition	6.4
$\delta^0$	subset of all cones along the unipotent fibre	6.5
$\delta (P_1, X_1)$	restriction to a boundary component	6.5
$\delta_{[\sigma]}$	induced cone decomposition	7.7
$\delta(f)$	cone decomposition associated to a piecewise linear convex rational function	5.20
$\delta_\Sigma^*, \delta'_\Sigma, \delta_\Sigma$	cone decompositions associated to $\Sigma$	9.12
$ \delta $	support of a cone decomposition	5.1
$\sigma^0$	interior of a cone	5.1
$\sigma_0$	standard cone	5.10, 8.5, 9.1
$\check{\sigma}$	dual cone	5.1
$[\sigma]$	double coset of cones	7.1
$\Sigma$	locally polyhedral subset	9.8
$T_\delta$	torus embedding	5.3
$T_\sigma$	affine torus embedding	5.2
$\tilde{T}_\sigma$	orbit in a torus embedding	5.2
$U$	weight $-2$ subgroup of $P$	2.1

$\mathcal{U}, \mathcal{U}(P_1, \mathcal{X}_1, P_f)$	covering of $M^{Kf}(P, \mathcal{X})(\mathbb{C})$	6.10
$\bar{\mathcal{U}}, \bar{\mathcal{U}}(P_1, \mathcal{X}_1, P_f)$	covering of $M^{Kf}(P, \mathcal{X}, \delta)(\mathbb{C})$	6.13
$V$	weight -1 subquotient of $P$	2.1
$V[d], U[d]$	standard $\mathbb{Z}/d\mathbb{Z}$ -modules	10.3, 10.7
$w$	"weight" of $S$	1.3
$W$	unipotent radical of $P$	2.1
$W_n M$	weight filtration	1.1
$X_d \rightarrow A_d \rightarrow \mathbb{A}_d$	universal family of abelian varieties	10.10, 11.16
$X_\delta$	relative torus embedding	5.5
$X^*(T)$	character group of a torus $T$	5.2
$\mathcal{X}^+$	open subset of $\mathcal{X}$ that maps to $\mathcal{X}_1$	4.11
$\mathcal{X}^*$	union of hermitian symmetric domain with all its rational boundary components	6.2
$Y_*(T)$	cocharacter group of a torus $T$	5.2
$Z(n)$	Tate Hodge structure	1.11
$\sim$	equivalence relation on $\mathcal{U}$ or $\bar{\mathcal{U}}$	6.10, 6.16
$\mathcal{A}$	the complex number	0.2
$[ , ]$	commutator	0.3
$[ \cdot p_f ], [ \varphi ]$	morphisms of mixed Shimura varieties	3.4, 6.2, 6.25, 9.25, 12.3, 12.4
$[ \cdot p_f ]^* \delta, \varphi^* \delta$	pullback of admissible cone decomposition	6.5

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