Something strange is going on with the prime numbers

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Outline

- Some questions on primes and why we (I) care
- The Riemann Hypothesis through lots of pictures
- Recent input into Number Theory from physics
- My work

Primes

- A prime number is an integer with exactly 2 divisors: 1 and itself.
- Ex: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, $\cdots$

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  - any number factorizes in a unique way as a product of primes:

$$\begin{aligned} 1264 &= 2 \times 2 \times 2 \times 2 \times 79 = 2^4 \times 79 \\ 5355 &= 3^2 \times 5 \times 7 \times 17 \end{aligned}$$

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- yes, yes: $(5,7),(40,42),(75,77),\cdots(5+35k,7+35k)$
- yes, twin prime conjecture :
  $(11,13),(17,19),(29,31),(41,43),(71,73),\cdots?\cdots$

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In general, questions involving few fixed primes are always easier, thanks almost solely to the Chinese Remainder Theorem (Sun Tzu, 3rd century CE).

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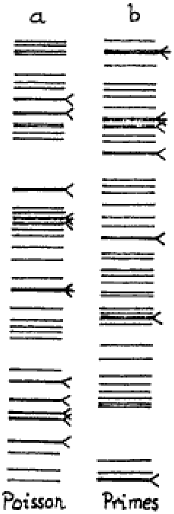
In contrast, many of the harder problems involve infinitely many primes, yet we still don't need more than addition and multiplication to state them.

Questions about primes (III)



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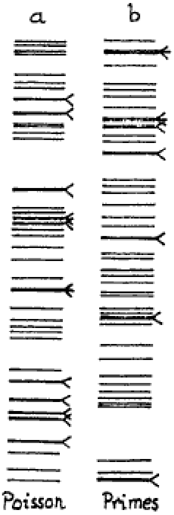

(Named for Siméon Poisson)

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- relevant to modern physics (in renormalization, $\cdots$)

Riemann Hypothesis

I will now introduce the Riemann zeta function in two different ways, and then present the Riemann Hypothesis, one of seven 1,000,000$ problems.

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\zeta(1) &= \infty
\end{aligned}$$

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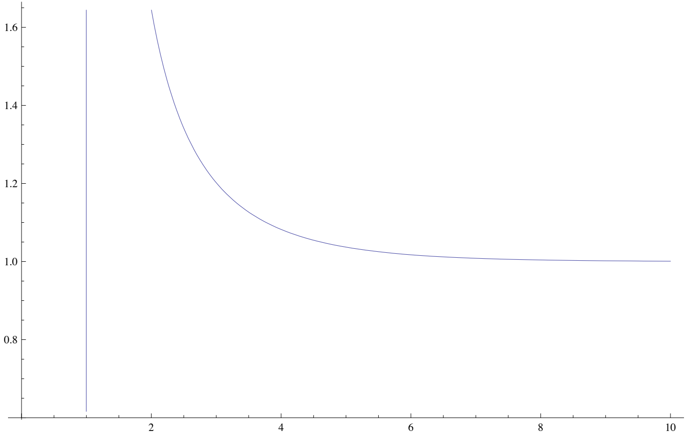

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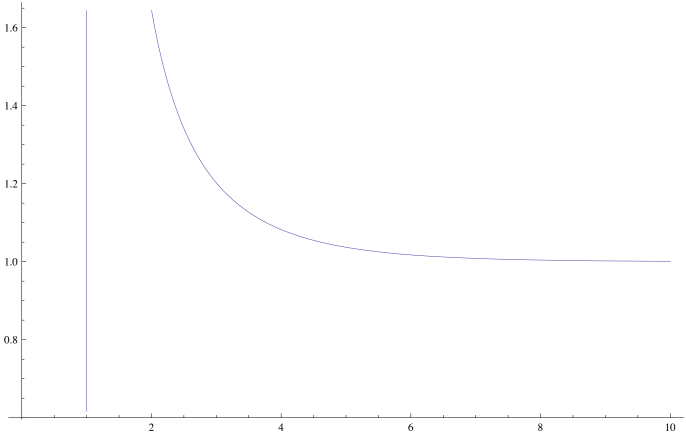
This perfectly encodes the unique factorization theorem!

Riemann zeta function

$$\zeta(x) = \sum_{i=1}^{\infty} \frac{1}{n^x} = \prod_{p \text{ prime}} \frac{1}{1 - p^{-x}}$$



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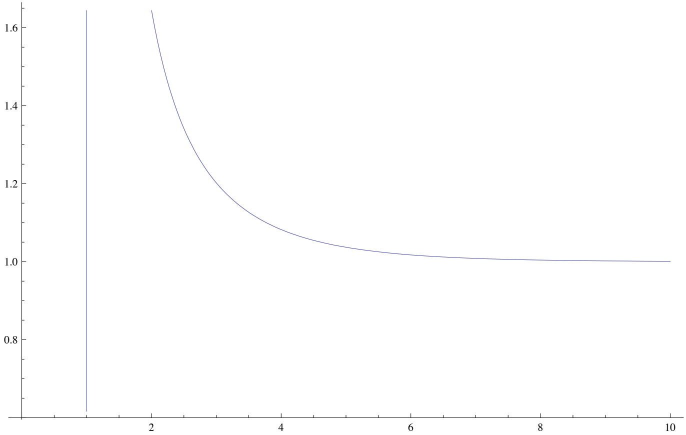
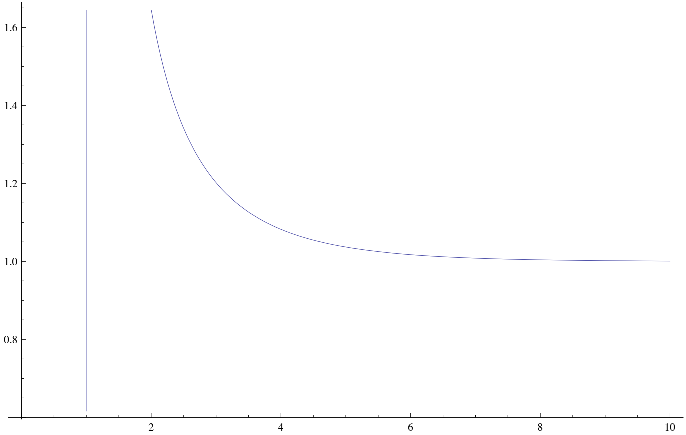


Remark: ζ involves all primes

Riemann zeta function

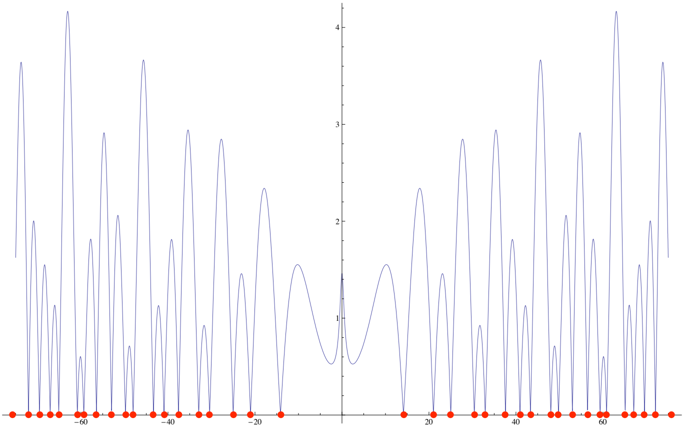
$$\zeta(x) = \sum_{i=1}^{\infty} \frac{1}{n^x} = \prod_{p \text{ prime}} \frac{1}{1-p^{-x}}$$



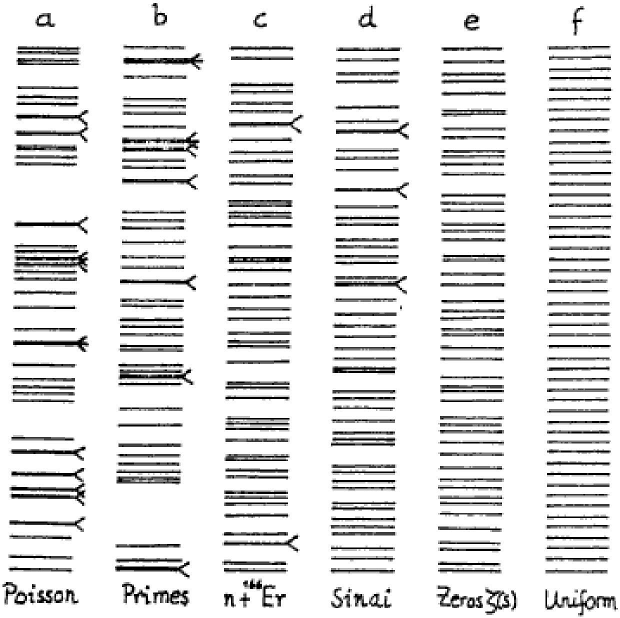
Remark: ζ involves all primes, and can actually be used to prove that there are infinitely many!

Movie of $|\zeta(s)|$

Along the critical line



Input from physics: zeroes



Neutron resonance

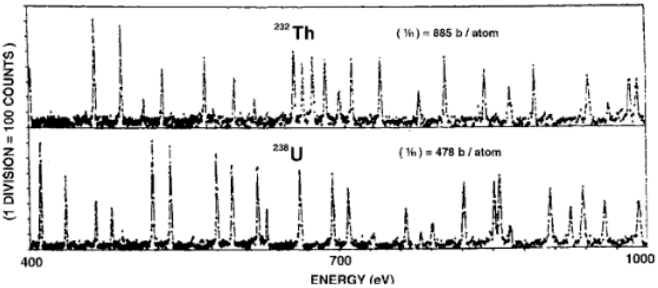
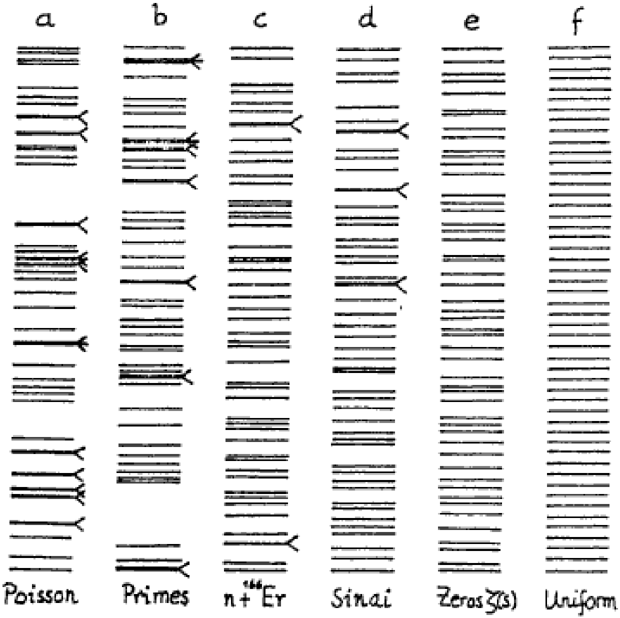
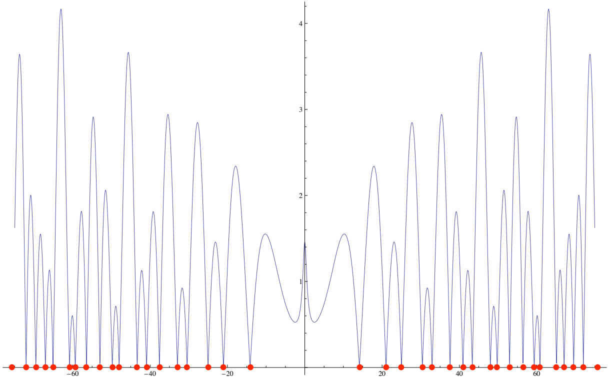


Figure 1.1. Slow neutron resonance cross-sections on thorium 232 and uranium 238 nuclei. Reprinted with permission from The American Physical Society, Rahn et al., Neutron resonance spectroscopy, X, *Phys. Rev. C* 6, 1854–1869 (1972).

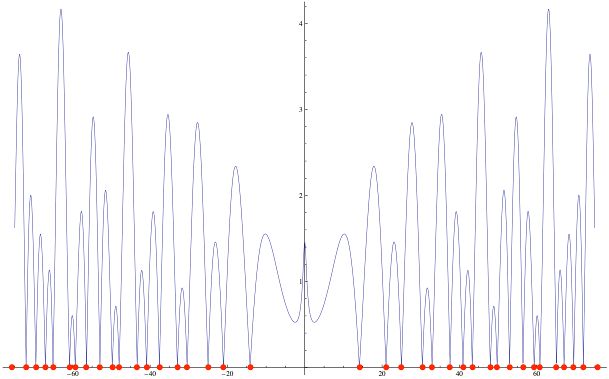
Input from physics: zeroes



Input from physics models: values

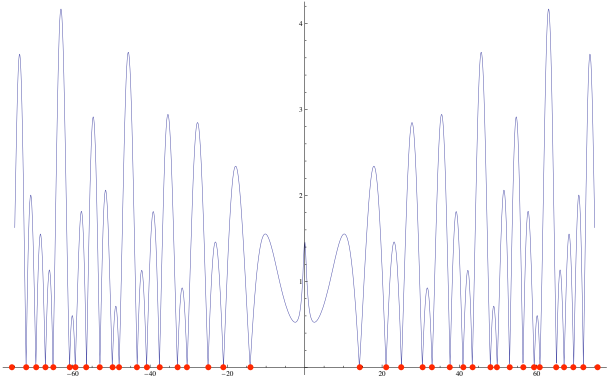


Input from physics models: values



Random matrices can be used to not only study the asymptotic distribution of zeroes of $\zeta(s)$ but also the values of $\zeta(s)$ along the critical line!

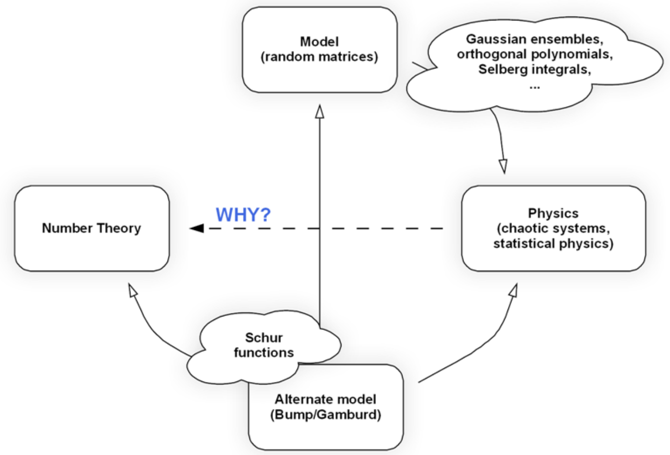
Input from physics models: values



Random matrices can be used to not only study the asymptotic distribution of zeroes of $\zeta(s)$ but also the values of $\zeta(s)$ along the critical line!

WHY?

My work


Model
(random matrices)
Gaussian ensembles,
orthogonal polynomials,
Selberg integrals,
...
Number Theory
WHY?
Physics
(chaotic systems,
statistical physics)
Schur
functions
Alternate model
(Bump/Gamburd)