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I have taught university-level mathematics for over ten years. This period covered different circumstances, locations and students, which have led me to change and re-change my methods and techniques of teaching over the years. These are thus not set, and future circumstances might alter them further. Nevertheless, one aspect of my teaching has evolved to a point where I think it will not change further, simply because it has worked for me. This concerns my personal motivation to teach well. Because of this conviction, I will describe my philosophy for teaching well. In a second part, I will detail the practical methods used to enact this goal. In the third part, I will list my various teaching accomplishments.

My teaching has been rewarded with the **Pólya Teaching Prize** at Stanford in 2006.

Why I want to teach well

Many universities or departments offer advice to their instructors, in multiple ways, on how to teach well. They also offer feedback tools, most often via student evaluations. All of these are valuable aids to the instructor, but they mostly focus on *how* to teach. Time and time again, mostly in the classroom or in discussions with with other instructors, both junior or senior to me, I have felt these tools can sometimes obscure the more fundamental question concerning mathematics teaching in universities: *why would a professional mathematician want to teach well?*

The answer to this question is necessarily personal, and I have found what works for me is the following principle.

I do mathematical research for the beautiful aha moments it can offer. I do mathematical teaching for the beautiful aha moments[1] I can offer.

This guideline is to be contrasted with other teaching strategies, that might focus on performance on the final exam, or the skills set presented to the students.

This guiding principle helps me clarify what my ideal one-hour lecture should be, helps me aim for it and do adjustments if needed. It comes with one potential pitfall, but many certain advantages. Let me address the pitfall first.

In some ways this principle is clearly egoistic: the risk would be to teach just for myself. To counteract this risk, an analogy with gift-giving works well: I want to offer beautiful mathematics to my students, and I need to make sure they realize how beautiful that present is! The average bachelor biology students would not see beauty where the average mathematics M.Sc. students would, so the class needs to be tailored for the students if I want them, and by extension me, to keep enjoying it. There is thus an inbuilt negative feedback loop that keeps me focused on the student perspective, and the pitfall becomes very easy to avoid.

For this one pitfall, my idealized class comes with many advantages. When the principle works well, it motivates me easily to give my best. It motivates the students as well, as they enjoy the class much more. By making aha moments a goal of each lecture, the class rewards students more directly for understanding the subject material and this fosters a deeper form of learning than is shown in traditional grades. It gives me more confidence in addressing the class (I do not

[1] I use this term by lack of a better one. What I mean is a distinct moment of clarity, where one feels their understanding is suddenly improved. In the case of mathematical research, this can come either from a discovery or from encountering someone else's idea.

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worry about student evaluations[2], the exam, etc). This emphasis lets me take more risks with my teaching methods (and thereby vary them more). It engages the students more and generally makes class more interactive (as the students *want* to get the mathematical punchlines). Finally, I find it helps me balance my time between research and teaching: my benchmark is that I need to enjoy both, and allow enough time for that. Much of the usual tension between these two tasks in my schedule is essentially voided.

Of course nothing is revolutionary in this approach. I only claim that it works for me, and has served me well in the past few years.

How I teach

For all of its advantages presented here, this principle still needs to be concretely implemented. Of course, my implementation first identifies those moments, big and small, where the students will understand something new. I then pace and structure the course around them.

Before the beginning of the term, I identify the main ideas the students need to understand in the course and the main mathematical problems they need to be able to solve[3]. I then try to develop central concepts into aha experiences the students might have. For this, I break up each idea or problem into two parts. The first part, which I explain at the very first lecture, tends to focus on questions the students would not know (yet!) how to answer (for non-math majors, I make sure to formulate this as a concrete problem in their area of interest). In the first lecture, I also promise the second part (the mathematical solution to the concrete problem) and announce it will be kept for later in the course. Generally the right time to deliver that punchline is right after the big theorem that covers the theory needed (my real target). This helps motivate the whole course, from the students perspective, and helps them grasp the power of the mathematics they are learning.

I try to structure each lecture in a similar way (of course at a different scale), with some smaller intermediate targets clearly presented to the students at the beginning of each lecture.

After each lecture, I spend an extra five minutes[4] reflecting on the completed lecture and figuring out what sparked their curiosity (judging from their questions) and which issues need to be addressed again. If an item is not critical to the next lecture, my preferred method is to turn it into a homework exercise, possibly due only a couple weeks later. This exercise will lead them to really think again about the material at their own pace, and force them to reread some of the materials from recent weeks. This helps the students memorize, structure and understand. By both design and convenience, all this makes for homework that span several weeks of material, overlapping from week to week, rather than covering just the current week of material. In those five minutes, I also think about what comes next in the course, so I can prime my brain to think about it more in my idle time leading to the next lecture. By the day before the next lecture, I usually have the detailed structure for that lecture figured out, and can just sit down and write it. I write in

[2]see later

[3]In contrast with other teaching methods, this decides the general material covered in the course *only*. Once that is decided, the general course and class structure follows the principles exposed here.

[4]generally the time it takes me to walk back to my office, as the ETH building is very big...

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my notes what I expect to write on the board (while I always end up adapting this a bit on-the-fly, I am not good at improvising additional material on the board). I also mark which statements I will need to back-reference, and therefore need to keep on the board for particularly long, since this might cause practical problems on the board during the lecture.

During the actual class, I pause often and make sure the students follow. In general, getting questions is not a problem, so it is not too hard for me to judge this aspect. I also tend to bounce questions back to the student(s), with hints, which helps make the class more interactive and helps me assess level of understanding. In any case, if I have made sure to pause for a relatively long time *before* my point (so as to ensure the students are actually listening to me then, and not just copying from the board), it is not too hard to judge if the students understand the point I am making from their body language. If I cannot tell, I will ask some questions to the whole class, which are usually answered by the same subset of students at first. If I am not convinced the other students are with me, I address more questions to the rest of the class, or I repeat my point, in a different way.

The final practical aspect of classes concerns the exams. My personal opinion (perhaps gained in Oxford) is that the roles of an examiner and an instructor would ideally be filled by different persons. Practicalities almost always prevent that, but it is still important that the students see me as an instructor before seeing me as an examiner. This generally means that I will not address questions on the details of the exam before a couple weeks prior to the exam (I will however promise to, and do, clarify those in due time).

Teaching accomplishments

I have taught in Zürich, Stanford, Oxford and Brussels. Student evaluations were taken at each of those universities but Oxford, and all the recent ones and some older ones can be obtained from my homepage[5].

I will now describe my teaching duties over the years, in reverse chronological order.

At ETH Zürich, 2008–present.

- Lecturer, in charge of all aspects of the course and sometimes supervising a teaching assistant, for a class size ranging from 10 to 30 students: *Introduction to polytopes*, *Analytic Theory of L-functions*, *Introduction to Analytic Number Theory, Enumeration Techniques and Hypergeometric Summation, Representation Theory of Finite Groups* and *Topics in Number Theory.*
- Supervisor for BSc. and MSc. theses for the following students: Karin Peter (*The LLL-algorithm and some applications*), Elena Widmer (*Shnirel'man density and Waring's conjecture*), Benjamin Berni (*An introduction to Szemerédi's theorem*), Meriton Ibraimi (*Integral Moments of elliptic curve L-functions over* $\mathbb{Q}$ and *The Hawkins random sieve*), Troy Koltes (*Soundararajan's bound for moments of the zeta function*), Patrick Kühn (*Selberg's central limit theorem*), Nicolas Wider (*Integrality of factorial ratios of height greater than 1*, in progress).

[5]On the first page of this teaching statement, I mentioned I was not worried about teaching evaluations. I do care about them, insofar that they provide feedback on the past class, useful to improve the next one.

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- Participated (unofficially) in the mentoring of Dirk Zeindler for parts of his Ph.D. thesis (*Associated class functions and characteristic polynomials on the symmetric group*, see joint paper).

At the University of Oxford, 2006–2008.

- Class tutor for the class *Lie groups*, Merton College, Michaelmas 2007.
- Conducted interviews for admissions to Merton College, 2007 and 2008.
- Ph.D. transfer thesis committee for Timothy Trudgian (*Gram's law fails a positive proportion of the time*).
- Class tutor for the *Number theory* class in the Stanford-in-Oxford program.

At Stanford University, 2001–2006.

- Pólya Teaching Prize, 2006.
- Teaching assistant, directly in charge of two sections of 20–45 students each: *Calculus I*, *Calculus II*, *Integral Calculus of Several Variables*, *Linear Algebra and Calculus of Several Variables* (over 1000 students in the class), *Ordinary Differential Equations*.
- Course assistant, in charge of 10–45 upper-class students: *Elementary Theory of Numbers*, *Matrix Theory and Applications*, *Linear Algebra and Matrix Theory II*, *Fundamental Concepts of Analysis*, *Analysis on Manifolds*, *Ordinary Differential Equations*.
- Putnam competition seminar mentor, 2004–2006.
- Graduate seminar co-organizer, 2002–2003.
- Private tutoring, 2001–2006.

At the Université Libre de Bruxelles, 1999 –2001. While an undergraduate, I served as student teaching assistant to the course *Groupes et symétries*. I held (along with graduate students or recent Ph.D.s) 3-hour exercise sessions for the first year bachelor students.