Wave Propagation in Unbounded Domains

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Spring Semester 2010

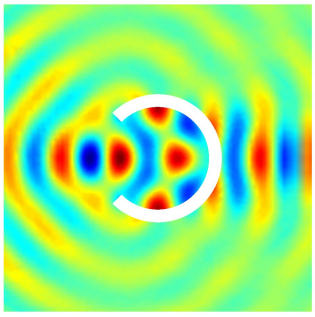
Tue & Fri 13-15 HG F 26.5

Abstract

This course is concerned with the fundamentals of scattering theory and the numerical simulation of wave propagation in unbounded domains.

Description

Wave propagation problems (imagine any wave!) are ubiquitous in Physics and Engineering. From a mathematical perspective, they pose interesting questions; existence and uniqueness proofs require the enforcement of physically sensible solutions in order to guarantee well-posedness. In particular, master wave equations (Helmholtz, elasticity, Maxwell's, etc.) do not discriminate between incoming or outgoing waves. Consequently, extra requirements in the form of radiation conditions need to be imposed. Accurate conditions allow theoretical proofs of existence and uniqueness while their numerical implementation can be very challenging.



Acoustic scattering from an annular obstacle.

Syllabus

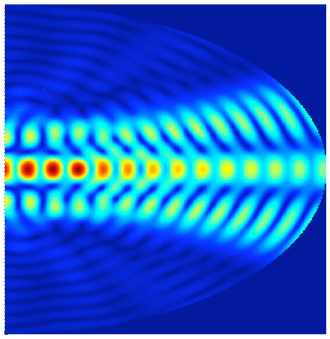
The preliminary syllabus is as follows:

- **Theory**:
  1. Wave physics in time dependent and time harmonic regimes [6, Chapter 1], [3, Section 2.1]:
     - Acoustic waves, Helmholtz equation
     - Electromagnetic waves
  2. Short review of mathematical tools [6, Chapter 2]
     - Norms, Hilbert spaces, sesquilinear forms, traces
     - Variational formulations
     - Coercivity, inf-sup condition, Gårding's inequality
     - Fredholm alternative
  3. Fundamental solutions. Green's theorem and formula [3, Section 2.2]
  4. Spherical Harmonics and Bessel functions [3, Section 2.3-4]

5. Radiation conditions:
   - Limiting absorption principle
   - Sommerfeld radiation conditions;
6. Existence and uniqueness of solutions for bounded domains.
   - Rellich's lemma
7. Far field mapping [3, Section 2.5]
8. Integral operators [7, 8]
   - Single and double-layer potentials
   - Boundary integral operators
   - Calderón projectors
9. The difficulty of solving problems with unbounded scatterers; Recent results [2, 1, 4]
10. Resonances

- **Numerics**

1. Modal expansions.
2. Review of FEM and BEM methods.
3. Non-local and Local Dirichlet-to-Neumann (DtN) maps [6, 5].
4. Perfectly Matched Layers (PMLs), Absorbing Boundary Conditions (ABCs).
5. On-Surface Radiation Conditions (**Invited lecturer Prof. X. Antoine**, Institut Elie Cartan Nancy, France)
6. Solving eigenvalue problems in unbounded domains.



Laser Gaussian beam propagating into free space.

Methodology and Evaluation

Course grades are given by an oral individual examination at the end of the semester.

References

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[8] J.-C. Nédélec. *Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems*, volume 144 of *Applied Mathematical Sciences*. Springer-Verlag, New York, USA, 2001.