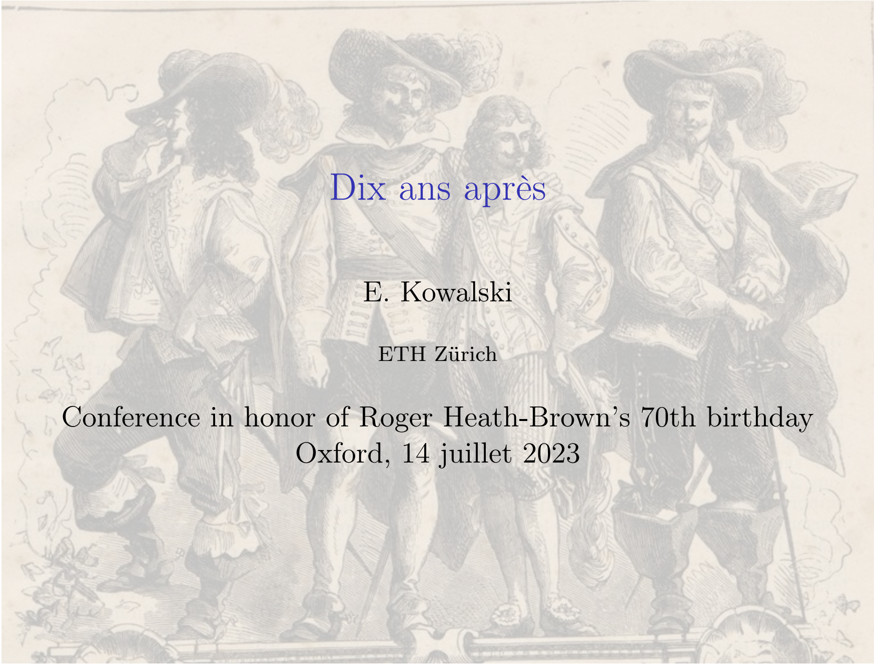


Dix ans après

E. Kowalski

ETH Zürich

Conference in honor of Roger Heath-Brown's 70th birthday
Oxford, 14 juillet 2023

Dix ans avant: Tuesday, September 4, 2012

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
0900–0930	*Registration*				
0930–1030	A. Gorodnik	R. de la Bretèche	B. Green	D. Masser	K. Soundararajan
1030–1100	*Coffee*	*Coffee*	*Coffee*	*Coffee*	*Coffee*
1100-1200	B. Conrey	H. Iwaniec	J. Brüdern	A. Kontorovich	G. Harcos
1200–1400	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
1400-1500	A. Granville	P. Michel	A. Cojocaru	C. David	
1500–1530	*Coffee*	*Coffee*	*Coffee*	*Coffee*	
1530–1630	P. Sarnak	E. Kowalski	M. Young	T. Wooley	
1700–1800	*Contributed talks* 1700–1730: F. Thorne 1730–1800: P. Vishe	*Contributed talks* 1700–1730: J. Van Order 1730–1800: A. Södergren		*Contributed talks* 1700–1730: C. Elsholtz 1730–1800: J. Stopple	
Evening	1830 : *Apéro* St. John's College		1900: *Conference drinks* 2000: *Conference dinner* New College		

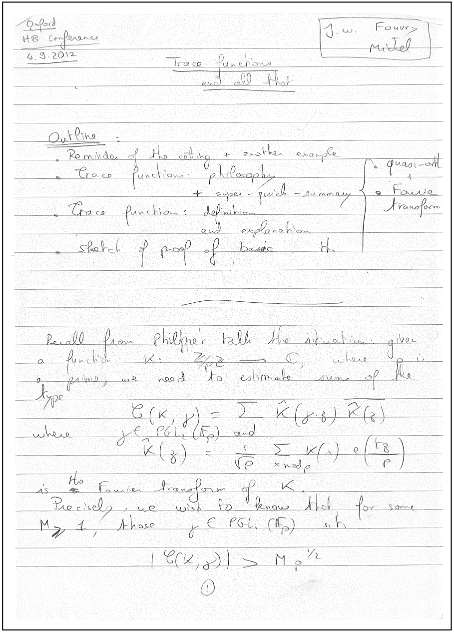
Philippe Michel, *Algebraic twists of modular forms, I*

Abstract: We consider estimates for sums of Fourier coefficients of modular forms twisted by functions of "algebraic origin". Using the amplification method and the Riemann Hypothesis over finite fields, in particular the Deligne–Laumon theory of the Fourier transform, we obtain very general estimates for such sums. This also has applications to the equidistribution of similarly twisted Hecke orbits. This is joint work with É. Fouvry and E. Kowalski.

Emmanuel Kowalski, *Algebraic twists of modular forms, II*

Abstract: This is the second part of a two-part talk shared with Philippe Michel.

Artefacts



Oxford
HB conference
4.9.2012

J. w. Fouvry
Michel

Trace functions
and all that

Outline:
- Reminder of the setting + another example
- Trace functions: philosophy
  + super-quick-summary
- Trace function: definition
  and explanation
- sketch of proof of basic th.

{ • quasi-orth. + • Fourier transform }

Recall from Philippe's talk the situation: given a function $K : \mathbb{Z}/p\mathbb{Z} \longrightarrow \mathbb{C}$, where p is a prime, we need to estimate sums of the type

$$\mathcal{C}(K,\gamma) = \sum \hat{K}(\gamma\cdot z)\,\overline{\hat{K}(z)}$$

where $\gamma \in PGL_2(\mathbb{F}_p)$ and

$$\hat{K}(z) = \frac{1}{\sqrt{p}} \sum_{x \bmod p} K(x)\, e\left(\frac{xz}{p}\right)$$

is the Fourier transform of K.

Precisely, we wish to know that for some $M \geq 1$, those $\gamma \in PGL_2(\mathbb{F}_p)$ s.t.

$$|\mathcal{C}(K;\gamma)| > M p^{1/2}$$

(1)

Trace functions

We recall examples of trace functions:

$$e\Big(\frac{f(x)}{p}\Big), \qquad \chi(f(x)), \qquad \frac{1}{\sqrt{p}}\sum_{x\in\mathbf{F}_p} e\Big(\frac{ax+f(x)}{p}\Big),$$

$$\mathrm{Kl}_k(x;p) = \frac{1}{p^{(k-1)/2}}\sum_{x_1\cdots x_k = x} e\Big(\frac{x_1+\cdots+x_k}{p}\Big),$$

$$\prod_{j=1}^{k}\mathrm{Kl}_k(a_jx;p)^{n_j}, \qquad \sum_{\substack{y\in\mathbf{F}_p\\ f(y)=x}} e\Big(\frac{ay}{p}\Big)$$

(here $f \in \mathbf{F}_p(\mathrm{X})$, $a \in \mathbf{F}_p$, χ is a multiplicative character)

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(here $f \in \mathbf{F}_p(\mathrm{X})$, $a \in \mathbf{F}_p$, χ is a multiplicative character) ... and recall that the set of trace functions is stable under sum, product, complex conjugation, *discrete Fourier transform*, additive or multiplicative convolution, etc, ...

The Riemann Hypothesis

Deligne's general form of the Riemann Hypothesis can be stated in the form of a "quasi-orthogonality" statement:

$$\frac{1}{p}\sum_{x\in\mathbf{F}_p} t_1(x)\overline{t_2(x)} = \nu(t_1,t_2) + \mathrm{O}\Big(\frac{c_1c_2}{\sqrt{p}}\Big),$$

where c_i is the *conductor* of the trace function t_i, and the main term $\nu(t_1,t_2)$ is of "algebraic nature", often easy to compute.

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$$\frac{1}{p}\sum_{x\in\mathbf{F}_p^\times} e\Big(\frac{ax+\bar{x}}{p}\Big) \ll p^{-1/2}$$

($t(x)=e((ax+\bar{x})/p)$ has conductor $\ll 1$ and is not constant).

A modest beginning...

In November 2011, Fouvry visited Philippe in Lausanne for two weeks and I was able to join them for a few days.
Fouvry was finishing a paper with S. Ganguly about sums like

$$\sum_{n\leqslant x}\lambda_f(n)\mu(n)e(n\alpha),$$

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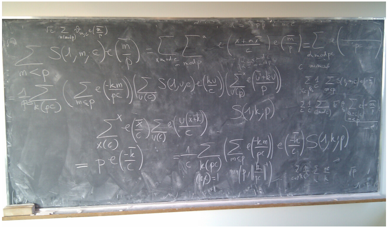

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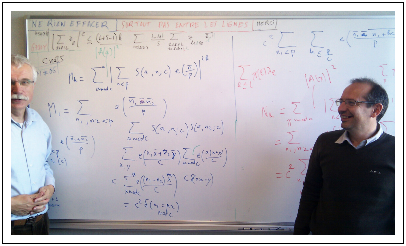

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$$\sum_{n \leqslant p} \lambda_f(n)e\Big(\frac{\bar{n}}{p}\Big).$$

By Jan. 2012, we had made the first positive progress.

FKM1 (2011-2012)

Our first paper proved strong orthogonality properties of trace functions against Hecke eigenvalues of classical modular forms:

$$\sum_{n \leqslant x} \lambda_f(n) t(n) \ll \mathsf{c}(t)^{10} x^{1-\delta} \qquad \text{(FKM1)}$$

if $p^{3/4+\varepsilon} \leqslant x \leqslant p^{\mathrm{A}}$, where f is a cusp form (of any level with trivial nebentypus).

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The proof of this result was the content of the talks of Philippe and I in 2012. This basic result was extended to squarefree moduli by B. Löffel (ETHZ), an archimedean analogue is due to A. Peyrot (EPFL), and a number field version to V. Nadarajan (EPFL).

FKM2

The second paper handled sums with the divisor function:

$$\sum_{mn \leqslant x} t(mn) \ll \mathsf{c}(t)^{10} x^{1-\delta} \qquad \text{(FKM2)}$$

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if $p^{3/4+\varepsilon} \leqslant x \leqslant p^{\mathrm{A}}$, unless t is an additive character; and sums over primes:

$$\sum_{n \leqslant x} \Lambda(n) t(n) \ll \mathsf{c}(t)^{10} x^{1-\delta}, \qquad \text{(FKM2)}$$

if $p^{3/4+\varepsilon} \leqslant x \leqslant p^{\mathrm{A}}$, unless t is the product of an additive character and a Dirichlet character.

The ternary divisor function (2012–2013)

The third paper exploited (FKM2) to give a streamlined proof, and improvement, of the exponent of distribution $> 1/2$ for d_3 in arithmetic progressions to prime moduli (first proved by Friedlander–Iwaniec, improved by Heath-Brown):

$$\sum_{\substack{n \leqslant x \\ n \equiv a \bmod p}} d_3(n) - \frac{1}{p-1} \sum_{n \leqslant x} d_3(n) \ll \frac{x}{(\log x)^{\mathrm{A}}} \qquad \text{(FKM3)}$$

for $p \leqslant x^{1/2+1/46-\varepsilon}$. (Recently further improved by P. Sharma, with $1/30$ instead of $1/46$.)

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This was the first "concrete" outcome of the project (submitted in April 2013).

A change of perspective

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For the first few months, we were working on a paper about *modular forms*, and applications. At some point, it also became a paper about *trace functions*, and especially about new ways of presenting Deligne's most general form of the Riemann Hypothesis over finite fields for *analytic* applications.

A crucial step (Feb. 28, 2012) was the introduction of the *conductor* of a trace function (or of the underlying algebraic object), as a unique invariant controling all(?) analytic estimates.

Answers

So it turned out that we ended up working to answer some general questions about exponential sums...

The profound theory of Deligne and other geometers is being used in analytic number theory with spectacular effects, yet more ideas need to be invented to fully exploit its potential.
(Iwaniec and K., chapter 11, page 315; 2004).

Finally, a vexing philosophical question: can one make [*Deligne's*] *theory "easier to apply"?*
(K., Milan J. of Math. 78; 2010).

Bounded gaps between primes

In 2013, the breakthrough work of Y. Zhang on gaps between primes relied at an essential point on the bound

$$\sum_{x\in\mathbf{F}_p^\times} \mathrm{Kl}_3(ax;p)\overline{\mathrm{Kl}_3(bx;p)}e\Big(\frac{hx}{p}\Big) \ll \sqrt{p}$$

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This was first made explicit in the Polymath 8A paper.

Sums of products (2013–2014)

More generally, sums of the type $\sum_{x \bmod p} t_1(x) \cdots t_k(x)$ occur everywhere in analytic number theory.

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Example. We have

$$\sum_{x \bmod p} \mathrm{Kl}_2(a_1 x + b_1; p) \cdots \mathrm{Kl}_2(a_k x + b_k; p) e\Big(\frac{hx}{p}\Big) \ll \sqrt{p},$$

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if either $h \neq 0$ or the (a_i, b_i) are pairwise disjoint modulo p. But

$$\sum_{x \bmod p} \mathrm{Kl}_3(x; p)^3 e\Big(\frac{hx}{p}\Big) = \delta(h)p + \mathrm{O}(\sqrt{p}).$$

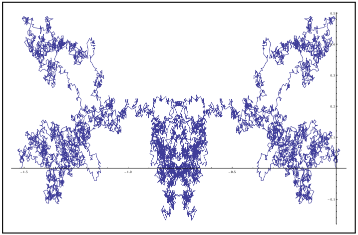
Enter Will Sawin

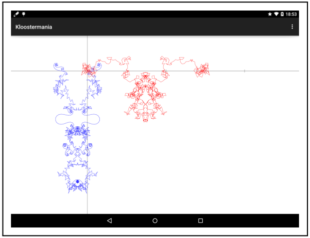
```
Date: Sun, 4 May 2014 21:49:54 -0400
From: wsawin@math.princeton.edu
To: emmanuel.kowalski@math.ethz.ch
Subject: Local theory of Integral Transform
User-Agent: Internet Messaging Program (IMP) H5 (6.1.3)

Hi Emmanuel,

I recently read your blog post "Conductors of One-Variable Transforms
of Trace Functions". I thought about the interesting question of
algebraic geometry you raised, and came to a rough answer. I thought
you might want to hear it, and it seemed a bit too long for a blog
comment, so I wrote this email about it:
(....)
```

Kloosterman paths (2014–2015)





Theorem. (K. and Sawin): the partial sums

$$\frac{1}{\sqrt{p}} \sum_{1 \leqslant x \leqslant (p-1)t} e\Big(\frac{ax + b\bar{x}}{p}\Big), \quad 0 \leqslant t \leqslant 1,$$

of Kloosterman sums "behave" like the random Fourier series

$$t\,\mathrm{ST}_0 + \sum_{h \neq 0} \frac{e(ht) - 1}{2i\pi h} \mathrm{ST}_h, \quad (\mathrm{ST}_h)_{h \in \mathbf{Z}} \text{ independent Sato–Tate.}$$

Bilinear forms with trace functions

Bilinear forms

$$\mathrm{B}_t(\alpha,\beta)=\sum_{m\sim \mathrm{M}}\sum_{n\sim \mathrm{N}}\alpha_m\beta_n t(mn),$$

for a trace function $t\colon \mathbf{F}_p \to \mathbf{C}$ also appear very frequently.

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for a trace function $t\colon \mathbf{F}_p\to\mathbf{C}$ also appear very frequently. The FKM2 paper gives an extremely general bound in the "Fourier" range:

$$\mathrm{B}_t(\alpha,\beta)\ll\|\alpha\|\,\|\beta\|\,(\mathrm{MN})^{1/2}\Bigl(\frac{1}{p^{1/4}}+\frac{1}{\mathrm{M}^{1/2}}+\frac{p^{1/4}(\log p)^{1/2}}{\mathrm{N}^{1/2}}\Bigr),$$

if t is not proportional to a product of an additive and a multiplicative character. This is non-trivial as long as N is a bit larger than $\sqrt{p}$ and M not bounded.

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$$\sum_{\chi \bmod p} |\mathrm{L}(f \otimes \chi, \tfrac{1}{2})|^2, \qquad f \text{ modular form}$$

we need $t(x) = \mathrm{Kl}_2(x; p)$ (Young in "Eisenstein case"; Blomer, K., Michel, Miličevič, Fouvry, Sawin).

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Theorem (K.–Michel–Sawin). There are bounds of this type in the case of $t(x) = \mathrm{Kl}_k(ax; p)$ and some hyper-Kloosterman sums with characters (in the sense of Katz).

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This is an order of magnitude harder (at least) than the previous results, and goes well beyond the "standard" formalism of étale cohomology. (For $k = 2$, alternative proof by Shkredov using additive combinatorics.)

Further applications

Other estimates for L-functions where bilinear forms appear include (unusual) “triple toroidal averages”:

$$\sum_{\chi \bmod p} \mathrm{L}(\tfrac{1}{2},\chi^a)\mathrm{L}(\tfrac{1}{2},\chi^b)\mathrm{L}(\tfrac{1}{2},\chi^c), \quad (a,b,c)=1,\ 1 \leqslant a \leqslant b \leqslant c$$

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The bounds of K.–Michel–Sawin are required here with $t(x)$ a Kloosterman sum in $a + b + c$ variables "with characters".

Interestingly, the theory only works if $a + b + c$ is *odd*. For instance, we don't yet see how to deal with

$$\sum_{\chi \bmod p} \mathrm{L}(\tfrac{1}{2}, \chi)\mathrm{L}(\tfrac{1}{2}, \chi^2)\mathrm{L}(\tfrac{1}{2}, \chi^3).$$

Related works, applications, etc

- Autissier–Bonolis–Lamzouri (distribution of large partial sums)
- Blomer–Miličevič (moments of L-functions)
- Bonolis (polynomial sieve, etc)
- Bourgain–Chang (non-linear Roth-type theorems)
- Darreye (distribution of coefficients of half-integral weight modular forms)
- Irving (bound for $L(\chi, \frac{1}{2})$ for χ mod q, q smooth, etc)
- Khan–Ngo (non-vanishing of $L(\chi, \frac{1}{2})$)
- Kunisky–Yu (properties of Paley graphs)
- Korolev–Shparlinski (twists by arithmetic functions)
- Mangerel (squarefree integers to smooth moduli)
- Nunes (distribution of squarefree numbers, etc)
- Perret-Gentil (short sums of trace functions, etc)
- Peyrot (archimedean analogue of FKM1)
- Polymath (exponential sums)
- Radziwiłł–Yang (non-vanishing of twists of L-functions on GL_4)
- Ricotta–Royer (Kloosterman paths modulo p^n)
- Shparlinski (and collaborators) (bilinear forms with various exponential sums)
- Xi (Katz's question on Kloosterman sums as Hecke eigenvalues, etc)
- Wu and Xi (general q-van der Corput)
- Zacharias (moments of L-functions)

(among others).