Jacques Ménard, author of Nicolas Bourbaki

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To Lucia

> Et dans ces grands livres-là, il y a des parties qui n'ont eu le temps que d'être esquissées, et qui ne seront sans doute jamais finies, à cause de l'ampleur même du plan de l'architecte.
>
> *And in these great books, there are parts that could only be sketched, and that will probably never be finished, because of the very ambition of the design of the architect.*
>
> Marcel Proust, *Le temps retrouvé*

The visible work left by this mathematician is easily and briefly enumerated. Impardonable, therefore, are the omissions and additions perpetrated by Prof. V.I. Siletzsky in a fallacious catalogue which a certain scientific society, whose Intuitionistic tendency is no secret, has had the inconsideration to inflict upon its deplorable readership. The true friends of Ménard have viewed this catalogue with alarm and even with a certain melancholy. One might say that only yesterday we gathered before his final monument, amidst the lugubrious cypresses, and already Error tries to tarnish his Memory... Decidedly, a brief rectification is unavoidable.

I am aware that it is quite easy to challenge my slight authority. I hope, however, that I shall not be prohibited from mentioning two eminent testimonies. Professors Rudnick and Sarnak (at whose unforgettable colloquia I had the honor of meeting the lamented geometer) have seen fit to approve the pages which follow. These recommendations, I think, are not entirely insufficient.

I have said that Ménard's *visible* work can be easily enumerated. Having examined with care his personal files, I find that they contain the following items:

a. The second-most rated answer to a *MathOverflow* question on the mathematical aspects of the detection of gravitational waves (February 2015).

b. A monograph on "certain connections or affinities" between the thoughts of Descartes, of Leibniz and of John Wilkins (Princeton, 2009).

c. A technical article on the possibility of improving the game of chess, by exchanging the position of the left rook and bishop. Ménard proposes, recommends, discusses and finally rejects this innovation.

d. A Go-playing computer program, written in Common Lisp.

e. A translation, with commentaries, of F. Enriques's book *Introduzione alla geometria sopra le superficie algebriche* (Paris, Ed. J. Gabay, 1995).

f. A manuscript of an elementary ergodic theory textbook.

g. An examination of the essential axioms of Talmudic discussion, with examples taken from the works of Rashi (Judaism, New-York, October 2012).

h. A reply to S. Cohen (who had denied the existence of such axioms), illustrated with examples from S. Cohen (Judaism, New-York, June 2013).

i. A long historical essay in the Catalogue of an exposition of periodic and non-periodic tilings (Bibliothèque de l'École normale supérieure, 2010).

j. A transposition into topos theory of the theory of Selberg's Sieve (Journal des Sciences Mathématiques, January 2005).

k. A note, published in the American Math. Monthly, concerning the resolution of the pythagorean equation by means of Hilbert's Theorem 90.

l. A note in the same journal acknowledging O. Taussky's priority concerning the result of the previous note.

m. An invective against K. Soundararajan, in the *Gaceta de la Real Sociedad Matemática Española*. (Parenthetically, this invective is the exact opposite of his true opinion of Soundararajan. The latter understood it as such and their old friendship was not endangered.)

n. Three articles, published in Math. Z., that discuss various aspects of the extension of the fixed-points theorems of Leray–Schauder to certain classes of bornological spaces.

o. An incomplete manuscript on the product of simply-connected spaces.

p. A manuscript list of mathematical formulas which owe their efficacy to their typography and notation.[1]

This, then, is the *visible* work of Ménard (with no omission other than a few vague problems or conjectures of circumstance written for the hospitable, or avid, problem-book of Prof. V.I. Siletzsky). I turn now to his other work: the subterranean, the interminably heroic, the peerless. And – such are the capacities of man! – the unfinished. This work, perhaps the most significant of our time, consists of the third and fourth chapters of the book of Integration of Nicolas Bourbaki's *Éléments de Mathématique* and a fragment of the second chapter of the book of General Topology. I know such an affirmation seems an absurdity; to justify this "absurdity" is the primordial object of this note.

Two texts of unequal value inspired this undertaking. One is a speculative short story whose author I do not remember which outlines the theme of a total identification with a given writer. The other is one of those parasitic books which make connections between Bourbaki and cubism or structuralism. Like all persons of taste, Ménard abhorred these useless carnivals, fit only – as he would say – to produce the plebeian pleasure of anachronism or (what is worse) to enthrall us with the elementary idea that all intellectual achievements are the same or are different. Those who have insinuated that Ménard dedicated his

[1] Prof. Siletzsky also lists a Russian translation of Hazewinkel's translation of Manin's "Cubic forms". There are no traces of such a work in Ménard's library. It must have been a jest of our friend, misunderstood by the professor.

life to writing a contemporary version of the *Éléments* calumniate his memory.

He did not want to compose another *Éléments* – which is easy – but *the* Éléments de Mathématique themselves. Needless to say, he never contemplated a mechanical transcription of the original; he did not propose to copy it. His admirable intention was to produce a few pages which would coincide – word for word and line for line – with those of Nicolas Bourbaki.

"My intent is simply astounding," he wrote to me the 30-th of September, 2011, from Princeton. "The final term in a geometric or arithmetic demonstration – Mostow's rigidity theorem, the undecidability of the Axiom of Choice, the classification of connected compact surfaces – is no less definitive than my book."

The first method he conceived was relatively simple. Know the personal history of all original members of Bourbaki, re-learn mathematics from the analysis treatise of Goursat, read the memoirs in Math. Annalen in chronological order, forget the history of mathematics since the year 1930, be Nicolas Bourbaki. Jacques Ménard studied this procedure (I know he attained a very creditable command of the rhetorical style common to memoirs of this period) but discarded it as too easy. Rather as impossible! my reader will say. Granted, but the undertaking was impossible from the very beginning and of all the impossible ways of carrying it out, this was the least interesting. To be, in the twenty first century, a reformer of the mathematical foundations dating to the twentieth seemed to him a diminution. To be, in some way, Bourbaki and reach the *Éléments* seemed less arduous to him – and, consequently, less interesting – than to go on being Jacques Ménard and reach the *Éléments* through the experiences of Jacques Ménard. (This conviction, we might say in passing, made him omit the historical notes to the *Éléments*. To include them would have been to present the *Éléments* in terms of Bourbaki's experience, and not of Ménard's. He naturally declined that facility.) "Essentially, my undertaking is an easy one," I read in another part of his letter. "I should only have to be immortal to carry it out." Shall I confess that I often imagine he did finish it and that I read the *Éléments* – all of them – as if Ménard had conceived them? Some nights past, while leafing through paragraph 1 of chapter II of the book of Functions of one real variable – never attempted by him – I recognized our friend's style and something of his voice in this exceptional remark: "One should not believe that ruled functions on an interval I are the only functions that admit an antiderivative on I." This happy conjunction of a familiar interjection and a truth reminds me of this line by Poe which we discussed one afternoon: "Ah! bear in mind this garden was enchanted!".

But why precisely the *Éléments de Mathématique*? our reader will ask. The letter already mentioned illuminates this point. "The *Éléments de Mathématique*," clarifies Ménard, "interest me deeply, but they do not seem – how shall I say it? – inevitable. I cannot imagine the universe without the Riemann zeta function, or without Brownian motion, or the hyperbolic plane, but I am quite capable of imagining it without the *Éléments*. (I speak, naturally, of my personal capacity and not of the historical resonance of these objects.) The *Éléments* form a contingent book; is is unnecessary. I can premeditate writing them, I can write them, without falling into a tautology. Between 15 and 24, I read them, perhaps in their entirety. Later, I have reread closely certain chapters, those which I shall not attempt for the time being. My general recollection of the *Éléments*, simplified by forgetfulness and indifference, can well equal the imprecise and prior image of a book not yet written. Once this idea (which no one can legitimately deny me) is accepted, it is certain that my problem is significantly more difficult than that of Nicolas Bourbaki. My obliging predecessors did not refuse collaboration and the most heated discussions to compose their work, which they wrote and re-wrote before arriving to a final redaction. I have taken on the mysterious duty of reconstructing alone and spontaneously their litteral and thoughtful work. My solitary game is governed by two polar laws. The first permits me to play with variations of a formal or psychological type; the second obliges me to sacrifice these variations to the "original" text and reason out this annihilation in an irrefutable manner... To these artificial hindrances, another – of a congenital kind – must be added. To compose the *Éléments* in the middle of the twentieth century was a reasonable undertaking, necessary and perhaps even unavoidable; at the beginning of the twenty-first, it is almost impossible. It is not in vain that seventy years have gone by, filled with mathematical discoveries and revelations. Among these, to mention only one, are the *Éléments de Mathématique* themselves."

In spite of these three obstacles, Ménard's fragmentary *Éléments* are more subtle than those of Nicolas Bourbaki. The latter opposes to the unrigorous doddering fictions of the *géomètres* and *analystes* of the 20th century the slow-moving and ponderous march of well-chosen axioms; Ménard selects as his mathematical world a barely emerging land of modern algebra and topology, void of categorical considerations. What a series of pointless classifications, generalizations and notational mayhem that selection would have suggested to D. Chaperon de Lauzières or to Dr. B. Krasuviecki! Ménard eludes them with complete naturalness. In his work there are no objects of the second kind, or great and small theorems. He neglects or eliminates definitions for definitions' sake. This disdain points to a new conception of mathematical treatises. This disdain condemns Littlewood, with no possibility of appeal.

It is no less extraordinary to consider isolated chapters. For example, let us examine paragraph 5 of chapter IV of the book of Integration, "Fonctions et ensembles mesurables". It is well known that Nicolas Bourbaki defines measurability as a property of approximation by functions of compact support. Nicolas Bourbaki had known F. and M. Riesz: his decision is understandable. But that the *Éléments de Mathématique* of Jacques Ménard – a contemporary of the Malliavin Calculus and of Marc Yor – should fall prey to such nebulous sophistries! V.I. Siletzsky has seen here an admirable and typical subordination on the part of the author to the Bourbakist ideology; others (devoid of insight), a transcription of the *Éléments de Mathématique*; Z. Rudnick, the influence of Grothendieck. To this third interpretation (which I judge to be irrefutable), I dare, with proper humility, add a fourth, which conforms very well with the almost divine modesty of Jacques Ménard: his resigned or ironical habit of propagating ideas which were the strict opposite of those he preferred. (Let us recall once more his rant against K. Soundararajan.) The texts of Bourbaki and Ménard are verbally identical, but the second is almost infinitely richer.

It is a revelation to compare Ménard's *Éléments de Mathématique* with those of Bourbaki. The latter, for example, wrote (Integration, Chapter III, paragraph 1):

> DÉFINITION 2. On apelle mesure (ou mesure complexe) sur un espace localement compact X, toute forme linéaire continue sur $\mathcal{K}(\mathrm{X}; \mathbf{C})$.
>
> Si μ est une mesure sur un espace localement compact X, la valeur de cette mesure pour une fonction $f \in \mathcal{K}(\mathrm{X}; \mathbf{C})$ s'appelle *l'intégrale de f par rapport à μ.*

Written in the early twentieth century, written by the polycephalic genius Nicolas Bourbaki, this definition is a formal endorsement of the functional approach to integration. Ménard, on the other hand, writes:

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>
> Si μ est une mesure sur un espace localement compact X, la valeur de cette mesure pour une fonction $f \in \mathcal{K}(\mathrm{X}; \mathbf{C})$ s'appelle *l'intégrale de f par rapport à μ.*

The integral as the *source* of the measure: the idea is astonishing. Ménard, a contemporary of P. Deligne, does not define measure as a property of a set, but as the process of averaging in a space of continuous functions. Measure, for him, is not what a set "weighs"; it is how functions interact in a global pattern with the underlying space. The final sentence is brazenly pragmatic.

The contrast in style is also vivid. The style of Ménard suffers from a certain affectation. Not so that of his forerunner, who handles with ease the conventions of mathematical writing of his time.

There is no exercise of the intellect which is not, in the final analysis, useless. A mathematical theory begins as a plausible description of platonic objects or of the universe; with the passage of time, it becomes a mere chapter – if not a paragraph or a name – in the history of science. The *Éléments de Mathématique* – Ménard told me – was, above all, an entertaining and useful book; now it is the occasion for patriotic toasts, pedantical insolence and obscene arrogant rejections. Fame is a form of incomprehension, perhaps the worst.

There is nothing new in these nihilistic considerations; what is singular is the determination Ménard derived from them. He decided to anticipate the vanity awaiting all man's efforts; he set himself to an undertaking which was exceedingly complex and, from the very beginning, futile. He dedicated his scruples and his sleepless nights to repeating an already extant book. He multiplied draft upon draft, revised tenaciously and tore up thousands of manuscript pages.[2] He did not let anyone examine these drafts and took care they should not survive him. In vain have I tried to reconstruct them.

[2] I remember his quadricular notebooks, his black crossed-out passages, his peculiar typographical symbols and his insect-like handwriting. In the afternoons he liked to go out for a walk on the hills around Zürich; he would take a notebook with him and make a merry bonfire.

I have reflected that it is permissible to see in this "final" *Éléments* a kind of palimpsest, through which the traces – tenuous but not indecipherable – of our friend's "previous" writing should be translucently visible. Unfortunately, only a second Jacques Ménard, inverting the other's work, would be able to exhume and revive those lost Troys...

"Thinking, analyzing, inventing (he also wrote me) are not anomalous acts; they are the normal respiration of the intelligence. To glorify the occasional performance of that function, to hoard ancient and alien thoughts, is to confess our laziness or our barbarity. Anyone should be capable of all ideas and I understand that in the future this will be the case."

Ménard (perhaps without wanting to) has enriched, by means of a new technique, the halting and rudimentary art of mathematical writing: this new technique is that of the deliberate anachronism and the erroneous attribution. This technique, whose applications are infinite, prompts us to read "The Theory of Functions" by Titchmarsh as if it were posterior to Donaldson's "Riemann Surfaces" and the book "Locales" of Prof. V.I. Siletzsky as if it were by Prof. V.I. Siletzsky. This technique fills the most placid works with adventure. To attribute "$\mathrm{SL}_2(\mathbf{R})$" to J-P. Serre or to John Milnor, is this not a sufficient renovation of its tenuous mathematical significance?

Translated, from the Spanish, by H.A.H.