CORRECTIONS FOR THE BOOK "ANALYTIC NUMBER THEORY" (COLL. PUBL. 53)

H. IWANIEC AND E. KOWALSKI

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Thanks to colleagues who have sent corrections, in particular F. Chamizo, B. Conrey, É. Fouvry, R. Heath-Brown, K. Kedlaya, J. Rouse, É. Royer, J-P. Serre and I. Shparlinski.

Introduction, Page 3, Line 6: Typo: "primary" should be "primarily".

Section 1, Page 19, Line -6: Replace roman M by $\mathcal{M}$ in three places.

Section 1, Page 29, Line 6: Write $\mathcal{M}_{f^2}(x)$ instead of $\mathcal{M}_f^2(x)$.

Section 3, Page 48, Line -7: The formula in Lemma 3.2 should have $\tau(\chi^\star)$ on the right-hande side instead of $\tau(\chi)$ as stated. The correct form is used in Chapter 12, page 324.

Section 3, Page 54, Line -11: The equal sign is misplaced in the statement of the quartic reciprocity law (3.56), which should read

$$\Big(\frac{\pi_1}{\pi_2}\Big) = (-1)^{(p_1-1)/4(p_2-1)/4}\Big(\frac{\pi_2}{\pi_1}\Big).$$

Section 3, Page 55, Line 7: The character χ should be $\chi_{\pi_1}^2\cdots\chi_{\pi_r}^2$ (i.e., the square of the one indicated).

Section 3, Page 62, Line 8: In the case $\mathfrak{m}=(1)$, i.e., if $4\mid\ell$, it should be remarked that the expression in (3.91) is independent of the choice of a generator α of $\mathfrak{a}$, since in that case there may not exist a primary generator.

Section 4, Page 78, Line -8: The Ramanujan sums are on the right side in (4.56).

Section 4, Page 79, Line 7: The correct quotation in **[GR]** is 8.451.2 instead of 23.451.2.

Section 4, Page 87, Line -13: In formula (4.81), the right-hand side should be

$$\frac{1}{Y}\Big(\frac{\sin \pi x Y}{\pi x}\Big)^2$$

instead of

$$\Big(\frac{\sin \pi x Y}{\pi x Y}\Big)^2.$$

Section 5, Page 99, Line -8: Replace the formula by

$$\sum_n n^{1/2}\lambda_f(n)\exp\Big(-\frac{2\pi n}{X\sqrt{q}}\Big) = X^2\varepsilon(f)\sum_n n^{1/2}\overline{\lambda_f(n)}\exp\Big(-\frac{2\pi n X}{\sqrt{q}}\Big).$$

Section 5, Page 105, Line -13: D. Lieman's name is misspelled Liehman here, and also on page 140, line 17, and page 141, line 13 (it is correctly spelled in the bibliography).

Section 5, Page 119, Line 13: The root number of $L(s,\chi)$, as proved in Theorem 4.15, is $i^{-\delta}\tau(\chi)/\sqrt{q}$.

Section 5, Page 124, Line 5: The bound for $L(1,\chi)$ is $L(1,\chi)\gg\Delta^{-\varepsilon}$.

Section 5, Page 130, Line 15: Here and on line 18 and 24, $(\alpha/|\alpha|)^{ik}$ should be $(\alpha/|\alpha|)^{k}$.

Section 5, Page 130, Line 24: The factor 2 should be 4 (we are counting integers instead of ideals).

Section 5, Page 130, Th. 5.36: The main term should be $2\frac{\beta-\alpha}{\pi}\operatorname{li}(x)$.

Section 5, Page 130, Line -7: In the Hint, x designates the variable for the characteristic function of $[\alpha,\beta]$, not the limit x in Theorem 5.36. Also, the approximation to the characteristic function on $[0,2\pi]$ should be in fact periodic of period $\pi/2$ instead of symmetric with respect to $x\mapsto x\pm\pi$.

Section 5, Page 133, Line 13: It is *not* the case that the p-factor of the Rankin-Selberg convolution $L(f\otimes g,s)$ is the "naïve" one when $p\mid [q(f),q(g)]$; this only holds if p divides only one of the two conductors. This can be checked using the local Langlands correspondance, in the spirit of the Remark on p. 138; the point is that the local 2-dimensional representations at p for f and g might be of the form ρ_1, ρ_2 where ρ_1 is a so-called Steinberg representation and $\rho_2=\chi\rho_1$ is a twist of ρ_1 by a multiplicative character. In that case, there is a basis (e,f) of the underlying 2-dimensional space on which inertia acts by

$$\rho_1(i)e=e,\qquad \rho_1(i)f=\chi^{-1}(i)f$$
$$\rho_2(i)e=\chi(i)e,\qquad \rho_2(i)f=f,$$

and then both $e\otimes f$ and $f\otimes e$ are invariant vectors of $\rho_1\otimes\rho_2$ under inertia, which implies that the local p-factor for $f\otimes g$ is of degree 2 in p^{-s} (whereas the naïve factor is of degree 1).

Section 5, Page 135, Line 10: There is no exceptional zero for classical modular forms, as shown by J. Hoffstein and D. Ramakrishnan (*Siegel zeros and cusp forms*, Int. Math. Res. Not. 1995, No.6, 279-308 (1995).)

Section 5, Page 137, Th. 5.41: Molteni's result requires an assumption of the type $\lambda_f(p)\ll p^{1/4-\delta}$ for some $\delta>0$. The implied constant in the convexity bound then further depends on δ.

Section 5, Page 144, Line 16: The error term in the sample asymptotic for $\psi(x,C)$ on GRH is worse than it should be, namely the proof gives

$$\psi(x,C)=\frac{|C|x}{|\mathrm{Gal}(L/K)|}+O\Big(\sqrt{x}(\log x)(\sqrt{|C|}(\log x^{[K:\mathbf{Q}]})+\frac{|C|}{|\mathrm{Gal}(L/K)|}\log|d_L|)\Big).$$

simply by using the obvious estimate

$$|c_\rho|\leqslant\frac{|C|}{|\mathrm{Gal}(L/K)|}\deg\rho,$$

instead of $|c_\rho|\leqslant\deg\rho$.

Section 5, Page 148, Cor. 5.49, 5.50: In both corollaries, the L-functions involved are assumed to satisfy GRH.

Section 5, Page 149, Line 3,-14,-12,-11,-10: The lower bound in Theorem 5.51 should be $q\geqslant e^{0.8g}$ so $q>2$. In the proof the Ramanujan-Petersson bound gives

$$\Big|\frac{L'}{L}(A,\sigma)\Big|\leqslant 2g\Big|\frac{\zeta'}{\zeta}(\sigma)\Big|$$

(since the L-function is of degree $2g$), then in line -12, replace $-\zeta'(\sigma)/\zeta(\sigma)$ by $+2\zeta'(\sigma)/\zeta(\sigma)$. Taking $\sigma=3$ gives that the right-hand side is >0.4, hence the result.

Section 5, Page 149, Line -9: Since $e^{1.6} < 11$, one can't argue directly from Theorem 5.51 as corrected. However, Mestre **[Mes]** has shown using the explicit formula that $q > 10^g$, which gives the stated result.

Section 6, Page 157, Line 16: The formula should be really $z = y^{1/s}$ instead of $z = y^s$ (the definition is repeated correctly in the statements of Theorem 6.1, Cor. 6.2 and in Section 6.4).

Section 7, Page 181, Line -8: The modulus of $\sum X(p, \nu)$ should be squared.

Section 7, Page 182, Line 22: The reference to (7.8) should be to (7.38) instead.

Section 7, Page 188, Line 2: The argument $\sqrt{mn}$ of the Bessel function on the right-hand side of the inequality should be N. On line 5, the value of x should be the upper bound $x = 2\pi N c^{-1}$.

Section 8, Page 208, Line 4: Replace "g is given by" with "G is given by".

Section 9, Page 231, Line 17: An exponent 2 is missing in $|c_\ell|$ which should be $|c_\ell|^2$.

Section 9, Page 231, Line -6: An exponent 2 is missing in $|a^{(1)}_{n_1} \cdots a^{(k)}_{n_k}|$ which should be $|a^{(1)}_{n_1} \cdots a^{(k)}_{n_k}|^2$.

Section 9, Page 233, Line 6: The closing parenthesis is missing and dt should not be subscripted.

Section 11, Page 277, Line 12: The factor d in $\sum_{rd=n} d \cdots$ should be removed (it is included in the sum over D that follows).

Section 11, Page 280, Line -2,-8: One should replace m and n by a and b respectively.

Section 11, Page 285, Line 6: Replace the formula by

$$\sum_{k \leqslant j < J} (r_j + s_j g) X^{(j-k)q} = 0$$

(i.e. change the k subscript to j in the sum).

Section 11, Page 287, Line 2: Replace the formula by

$$N_1 = q - N_0 - N_2 > \frac{q}{2} - 4m\sqrt{q}$$

(i.e. replace the second N_1 by N_2).

Section 11, Page 305, Definition, l. -16: Replace "are algebraic integers" by "are algebraic numbers".

Section 11, Page 308, Line -16: Remove "only" in "only all the...".

Section 11, Page 309, Line -14 to -9: Replace $K_r(\cdot, p)$ by $K_r(\cdot, q)$ (in six places altogether).

Section 11, Page 309, Line -11: Replace $\mathbb{F}_p$ by $\mathbb{F}_p^\star$.

Section 11, Page 309, Line -11: Replace $K_r(a_0 v^2, p)$ by $K_r(a_0 v^r, q)$.

Section 12, Page 327, Line -1: The exponent of V_1 should be $1 - \frac{1}{r}$ instead of $1 - \frac{1}{4}$.

Section 14, Page 358, Line 13: The first factor in the formula for $p(m, n)$ should be $(n/m)^{(k-1)/2}$ (which is implicitly used in the proof of the Petersson formula).

Section 14, Page 359, Line 14: Although the "inner-product" is well-defined when f is in $M_k(q, \chi)$, the formula for $\langle f, P_m \rangle$ is only valid if f is a cusp form (as is the case in all applications in the book).

Section 14, Page 374, Line 9: The factor p^{-s} is missing in the Euler product:

$$L(f, s) = \prod_p (1 - \lambda_f(p) p^{-s} + p^{k-1-2s})^{-1}$$

Section 14, Page 375, Line 15: Same correction as the previous one.

Section 15, Page 399, Line -6: In the integral, $k(i,z)$ should be replaced by $k(u(i,z))$.

Section 16, Page 413, Line -4: The correct quotation in [**GR**] is 8.411.4 instead of 23.411.4.

Section 17, Page 421, Line 6: The bound on the right-hand side for the Barban-Davenport-Halberstam theorem should be $x^2(\log x)^{-A}$ instead of $x(\log x)^{-A}$.

Section 19, Page 448, Line 7: The N^3 in the error term should be N^2.

Section 20, Page 479, Line -12: $\delta_2(n,Q)$ instead of $\delta_2(n.Q)$.

Section 21, Page 487, Line 8: The open sets B for which equidistribution is tested should be such that the boundary of B in X has measure 0.

Section 21, Page 490, Line 9: The Weyl criterion reads $S = o(w(p)^n)$ instead of $S \ll w(p)^n$.

Section 21, Page 492, Line -5: The Sato-Tate measure should be $2\pi^{-1}\sin^2\theta d\theta$.

Section 22, Page 512, Line 13: The correct quotation in [**GR**] is 8.451.6 instead of 23.451.6.

Section 26, Page 587, Line 13: The correct quotation in [**GR**] is 6.561.14 instead of 14.561.14.

Bibliography, Page 605, Line 6: The reference for the paper of Kim and Sarnak should be [**KiS**], as quoted in Chapters 5 and 15.

Bibliography, refs. **[W]**: There are two references [**W**], one on page 609 (Washington's "Cyclotomic fields", quoted on page 67 and 68) and one on page 610 (Wiles's paper on Fermat's Great Theorem, quoted on page 145 and page 366).