

# A brief panorama of mathematics in Switzerland

By Max-Albert Knus

**T**he eighteenth century, the century of the Bernoullis and Euler, is the golden age of Swiss mathematics. The Bernoullis, a wealthy family of Basle, brought forth eight mathematicians in three generations. Together with Euler, they developed the calculus of Leibniz and Newton into the practical tool for physics and mechanics which we still use today. The first two, the brothers Jakob I (1654-1705) and Johann I (1667-1748) are the most famous mathematicians of the family. Jacob is considered to be the founder of the calculus of variations. He also made important contributions to probability theory. Johann's strength lay in the development



13 Sir Isaac Newton

of the more formal aspects of calculus, especially of integral calculus. Jacob was professor at the University of Basle from 1687 (when he was twenty years old) until his death in 1705. This university, with a tradition going back to the fifteenth century, was the only university in Switzerland. Johann first studied medicine but soon switched to mathematics. A professor in Groningen at the time, ten years later he was the successor to his brother in Basle. Daniel (1700-1782), son of Johann I, had a universal mind. Professor in St. Petersburg, where he became a great friend of Euler, and

later in Basle, he successively taught mathematics, philosophy, natural sciences and medicine. The family soon became a legend. The following anecdote is quoted in Bell's "Men of Mathematics". On a journey, after introducing himself to a foreigner, Daniel once got the sarcastic rejoinder, "and I am Isaac New-

ton". For Daniel this was a wonderful mark of esteem.

## Students of the Bernoullis

Many mathematicians of the next generation were in some way students of the Bernoullis. This was the case of Gabriel Cramer (1704-1752), professor at the Academy in Geneva, and still known for "Cramer's Rules" for solving linear equations. But the most famous student of the Bernoullis was Leonhard Euler (1707-1783). Also born in Basle, he studied theology and later mathematics with Johann, who even gave him private lessons in medicine. The lack of positions in Basle persuaded Euler to leave for St. Petersburg in 1727 to join Daniel and Nicolas Bernoulli. In 1741, after a difficult time in Russia, he accepted the invitation of Frederick the Great to join the Academy in Berlin. Called back to St. Petersburg in 1766 by Catherine II, he "ceased living and computing" (Condorcet) on September 18, 1783. Euler's name is present everywhere in mathematics. Every day mathematicians work with Euler's equations, integrals, numbers, angles, ... and use Euler's theorems! In 1909 the Swiss Association for Natural Sciences decided to publish Euler's complete

works and received the support of people and societies all over the world. Most of Euler's mathematical papers are now published, but it may take another ten years to complete the publication of his correspondence.

## Proof of the irrationality

Another Swiss mathematician who played a significant role outside Switzerland was Johann Heinrich Lambert (1728-1777). Born in Mulhouse into a protestant family of French origin, he worked first as a secretary for a Basle lawyer and from 1746 as a private tutor. From 1756 to 1764, he travelled around most of Europe, first as a companion to his pupils, then on his own. In 1764 he stopped in Berlin, hoping to be elected a member to the Academy of Sciences. His wish was fulfilled a year later and he even got an influential position in the Prussian civil service. An autodidact par excellence, Lambert was one of the typical universal minds of the eighteenth century and wrote many essays on astronomy, logic, mathematics, philosophy and art. In mathematics, he is mainly recognized for his proof of the irrationality of Pi. At the turn of the eighteenth century, one finds another Swiss autodidact: Robert Argand (1768-1822).



14 Leonhard Euler



15 Jakob Bernoulli



16 Johann Bernoulli



17 Johann Heinrich Lambert

Born in Geneva, he spent his life in Paris as a simple bookkeeper and a passionate amateur mathematician. He was one of the first to give a clear description of the sum and the product of complex numbers (1806). He was also the first to formulate the “fundamental theorem of algebra” for polynomials with complex coefficients (1815). His proof, known today as the “Argand-Cauchy” proof, is, however, not completely rigorous.

### Synthetic Geometry

Two names dominate the first half of the nineteenth century: Steiner (1796-1863), the geometer, and Sturm (1803-1855), the analyst. Like Euler and Lambert, they had to leave Switzerland to pursue mathematical careers. The son of a farmer, Jacob Steiner learned to read at the age of 14 and left his family when he was 18 to join



18 Jacob Steiner

Pestalozzi’s school in Yverdon. This famous pedagogue (1746-1827) quickly recognized the gifts of the young man and sent him, with a strong recommendation, to study mathematics at Heidelberg. With no other resources, Steiner eked out his meager existence by giving private lessons. After five semesters in Heidelberg, he left for Berlin, without a degree, to complete his studies. Two things kept him there: a job at the “Gewerbeschule” and the new Crelle Journal for Mathematics, in which he got the chance to publish his first papers. In 1834 Steiner was elected to the Berlin Academy of Sciences after being nominated by Crelle and Dirichlet and, in the same year, was appointed associate professor at the University of Berlin. Steiner worked mainly in synthetic geometry. In particular, he was the first to present the foundations of projective geometry in a systematic way. His health seriously affected, he spent the winter 1862/63 in Berne and died April 1, 1863, after having obstinately refused the help of a doctor.

### Zeros of the real polynomials

Charles Edouard Sturm was born in Geneva. His father, a strict and methodic man, made a modest living by teaching arithmetic. He died when Sturm was 16. From 1818 to 1823, Sturm studied at the Academy in Geneva and got his first position as tutor to the brother of Mme de Broglie, the youngest son of the celebrated Mme de Staël. The de Broglie family was living at the time in the Château of Coppet, near Geneva. When they returned to Paris, Sturm took the opportunity of visiting the French capital and, within a few months, met the best French math-

ematicians and physicists of the epoch. He moved to Paris permanently in 1825. Some notes which appeared in the “Annales de Gergonne” attracted the attention of Fourier. This great mathematician liked to help beginners and initiated Sturm into his recent work, thus opening up new mathematical horizons to this still young man. Sturm was barely 26 when he presented his famous theorem on the zeroes of real polynomials to the French Academy. Also well known is the “Sturm-Liouville” theory of differential equations of the second order. Begun in 1836 by Sturm, the theory was pursued later by his friend Liouville. After some years of teaching, Sturm became mathematics professor at the famous Ecole Polytechnique and in 1840 also took the Chair of Mechanics at the Sorbonne.

### Theory of manifold continuity

Ludwig Schläfli (1814-1895) is another interesting personality of the nineteenth century. Like other mathematicians of this period he first studied theology at the newly created University of Berne. A teacher in Thun, he devoted the little spare time (and money) he had to the study of higher mathematics. In 1843 Schläfli was introduced to Steiner. Invited by the great geometer, Schläfli followed Steiner to Rome after obtaining a loan, where, together with Borchardt, Jacobi and Dirichlet, he spent the winter 1843/44. Schläfli had a great gift for languages and served as an interpreter for his companions. He even translated



19 Ludwig Schläfli

papers of Steiner and Jacobi into Italian. In return, he received private lessons in the mornings, in particular from Dirichlet in algebraic number theory. Back in Berne, Schläfli had to work years as a poor "Privat-Dozent" before being elected a professor. His most original work, his "Theory of manifold continuity" was submitted for publication in 1851 to the Academy of Sciences of Vienna, but the paper was rejected because of its length and did not appear until after the death of the author. Schläfli presented abstracts of his theory in the "Journal de mathématiques pures et appliquées" (1855) and in the "Quarterly Journal of Mathematics" (1858). The main re-

sults concern generalizations of Euclidian geometry to  $n$ -dimensional space. Unfortunately, the papers were (and still are) hard to read. Many results were re-discovered later by other mathematicians and presented in more accessible language. Before 1850 Switzerland hosted only three universities: Basle, founded in 1460, and Zürich and Berne, founded in 1833 and 1834 respectively. The situation changed completely with the creation of the Swiss Institute of Technology (ETH) in 1855, the transformation of the Academies of Geneva (1874), Lausanne (1890), Neuchatel (1909) into universities and the creation of the University of Fribourg in 1889.



20 Richard Dedekind



21 Daniel Bernoulli

## The ETH

Raabe (1801-1859), the first professor of mathematics at the ETH, also held a professorship at the University. He resigned for reasons of ill health in 1857. His name is attached to a convergence criterion for series of positive numbers. His successor was Richard Dedekind (1831-1916). A full professor at the age of 27, Dedekind left six years later for Braunschweig, his native city. The definition of real numbers as "Dedekind cuts" dates back to November 1858. Dedekind was preparing his first calculus course when he was confronted with the difficulty of getting a useful definition of continuity (see the foreword of "Stetigkeit und irrationale Zahlen"). The ETH was very grateful to Dedekind for his help during its first years and, after obtaining the right to confer Ph.D.s in 1909, the school presented him with its first honorary degree in mathematics.

## Guests

Dedekind was the first of an impressive sequence of brilliant young German mathematicians, who got their first permanent academic position in Switzerland and later went back to Germany as well-established professors. Examples are Edwin Bruno Christoffel (1829-1900), Emil Pryn (1841-1915), Hermann Amandus Schwarz (1843-1921), Heinrich Weber (1842-1913), Ferdinand Georg Frobenius (1849-1917), Friedrich Schottky (1851-1935), Hermann Minkowski (1864-1909) and Hermann Weyl (1885-1955) at the ETH, Erhard Schmidt (1876-1959), Ernst Zermelo (1871-1953) at the University of Zürich and Erich Hecké (1887-1947) in



22 Ferdinand Georg Frobenius

Basle. The ETH had a position for mathematics in French and attempts were made to attract promising young French mathematicians. For example, in 1882, Emile Piccard (1856-1941) and Henri Poincaré (1854-1912) were approached but, unfortunately, without success.

We conclude this short account by recalling the names of some other mathematicians who were active in this century in Switzerland: Adolf Hurwitz (1859-1919), Rudolf Fueter (1880-1950), Michel Plancherel (1885-1967), Andreas Speiser (1885-1970), Georg Polyà (1887-1985), Paul Bernays (1888-1977), Alexander Ostrowski (1893-1986), Paul Finsler (1894-1970), Heinz Hopf (1894-1971), Rolf Nevanlinna (1895-1980), Georges de Rham (1903-1990), Hugo Hadwiger (1908-1981), Eduard Stiefel (1909-1978), and Martin Eichler (1912-1992).



23 Hermann Weyl