# Ausgewählte Themen der Funktionentheorie

The aim is to present selected topics from the theory of analytic functions of one variable. After some preparatory material related to the maximum modulus theorem (see Titchmarsh, chapter V) we will concentrate on the following two topics.

• Hypergeometric Functions:

Hypergeometric functions are solutions of second order differential equations on the complex plane with three regular singular points. There are series and contour integral representations for these functions, which we shall study. Many special functions can be viewed as special cases or degenerations of hypergeometric functions, for example Bessel functions, Legendre functions or elliptic integrals.

• Boundary values of holomorphic functions on the upper half plane:

A typical example is the Nevanlinna–Herglotz theorem that a holomorphic function $f(z)$ on the upper half plane with non negative imaginary part can be represented in the form

$$f(z) = b + cz + \frac{1}{\pi} \int_{-\infty}^{\infty} \left[ \frac{1}{t-z} - \frac{t}{1+t^2} \right] d\mu(t)$$

with $b \in \mathbb{R}$, $c \geq 0$ and a Borel measure $\mu$, and the ways $\mu$ can be reconstructed from $f$. In general, we will consider classes of functions and distributions that arise as boundary values of holomorphic funtions on the upper half plane (for example Hardy functions).

Some applications in physics (like the theorem of Baym and Mermin about analytic continuation of Green's functions in solid state physics or properties of special one dimensional Schrödinger operators) and in number theory (like bounds on the Riemann zeta function on the critical line) will be mentioned.

**Time:** Tuesday 15-17 in HG D7.1 and Friday 9-10 in HG F3.

The course starts on Tuesday, September 20

**Exam:** Oral exam of 20 min.

**Literature:**

E.Titchmarsh: The Theory of Functions. Oxford University Press

R.Remmert: Funktionentheorie II. Springer Verlag
(German and English Version online at ETH library)

M.Rosenblum, J. Rovnyak: Topics in Hardy Classes and Univalent Functions
(online at ETH library)

A.Gogolin:Komplexe Integration. Springer Verlag
(German and English Version online at ETH library)

C.Caratheodory: Funktionentheorie. Birkhaeuser

E.Hille: Analytic Function Theory. AMS Chelsea Publishing

F.Klein: Vorlesungen über die hypergeometrische Funktion. Springer

M.Yoshida: Fuchsian Differential Equations. Vieweg

H.Dym, H.McKean: Fourier Series and Integrals. Academic Press

J.Garnett: Bounded Analytic functions. Springer

G.Teschl: Ordinary Differential Equations and Differential Systems.
American Mathematical Society

O.Perron: Die Lehre von den Kettenbrüchen II. Teubner

L.Landau, E.Lifschitz: Quantenmechanik (Mathematische Anhänge)

A.Fetter, J.Walecka: Quantum Theory of Many–Particle Systems. Mc Graw–Hill

A.Fetter, J.Walecka: Quantum Theory of Many–Particle Systems. Mc Graw–Hill

K.Tomita: Plane symmetric vacuum solutions with null singularities for inhomogenuous models and colliding gravitational waves. arXiv:gr-qc/9807033