Workshop „Mathematical Finance beyond classical models“

September 16 – 18, 2015

Semper Aula HG G 60

Department of Mathematics

ETHZ

Abstracts (in order of appearance)

Josef Teichman (ETH Zürich)

"Tractable American Option Problems"

Following ideas of Soren Christensen, Benjamin Jourdain, Jan Kallsen and Claude Martini we analyse the question which American options can be statically hedged by European options and which multivariate numerical techniques follow from such relationships. Mathematically speaking this is related to analyzing the range of maximal functionals of analytic semigroups (joint work with Oleg Reichmann).

Thorsten Schmidt (University of Freiburg)

"Dynamic Term Structure Modelling beyond the Intensity Paradigm"

We extend the Heath-Jarrow-Morton approach to bond markets by allowing for events which take place at predictable times. This introduces a discontinuity in the term structure which requires a suitable generalization of the setup. It turns out, that previously considered models lead to arbitrage possibilitie. The chosen generalization of the forward-rate approach contains an additional stochastic integral with atoms at predictable times and necessary and sufficient conditions for an appropriate no-arbitrage condition (NAFL) are given. In the view of efficient implementations we develop a new class of affine models which do not satisfy the standard assumption of stochastic continuity. Examples include interest rate models and credit risky models.

Mathieu Rosenbaum (Université Pierre et Marie Curie)

"Asymptotic Lower Bounds for Optimal Tracking: a Linear Programming Approach"

We consider the problem of tracking a target whose dynamics is modeled by a continuous Ito semi-martingale. The aim is to minimize both deviation from the target and tracking efforts. We establish the existence of asymptotic lower bounds for this problem, depending on the cost structure. Theses lower bounds can be related to the time-average control problem of Brownian motion, which is characterized as a deterministic linear programming. A comprehensive list of examples with explicit expressions for the lower bounds is also provided.
This is joint work with Jiatu Cai and Peter Tankov.

Meter Soner (ETH Zürich)

"Trading with Market Impact"

We consider a financial market in which our trading causes price impact and portfolio optimization in such markets. Resilience is also used. The classical dynamic programming equation approach provides equations that we analyse in several different asymptotic regimes.

Yan Dolinsky (Hebrew University Jerusalem)

“Limit Behaviour of Super-replication Prices with Small Transaction Costs: Multi--asset Version of Kusuoka Results”

We consider super--replication with small transaction costs in a complete multi--asset, multinomial markets. For this setup we prove that the limit of the corresponding super--replication prices equals to a multidimensional G--expectation which we find explicitly. These results can be interpretated as the multidimensional extension of Kusuoka (1995). Joint work with P.Bank and A.P.Perkkio.

Marcel Nutz (Columbia University New York)

“Optimal Transport and Robust Finance”

After a brief introduction to classical optimal transport, we shall focus on the so-called martingale optimal transport and its connection to finance, the problem of robust semi-static hedging. Some differences with the classical transport problem will be highlighted, in particular the failure of duality in the usual sense. We explain how to obtain a complete duality theory using notions related to Knightian uncertainty about pricing models. Based on joint work with Mathias Beiglböck and Nizar Touzi.

Antoine Jacquier (Imperial College London)

“Martingale Information of the Implied Volatility Smile”

Given an implied volatility smile at some fixed maturity, we wish to answer the following questions regarding the underlying stock price process:
- Is it a true martingale?
- Does it have a strictly positive mass at zero?

We shall see that the answers to these two questions can be quantified precisely in terms of the tails of the implied volatility smile. In doing so, we shall revisit some of the model-free asymptotic behaviours of the implied volatility smile developed by Roger Lee and Shalom Benaim and Peter Friz. These results also provide a new testable characterisation of bubbles.
This talk is based on joint works with Stefano de Marco and Caroline Hillairet, and Martin Keller-Ressel.

Christa Cuchiero (University of Vienna)

“Polynomial Processes in stochastic portfolio theory”

We consider polynomial processes on the unit simplex which can be used as models for the market weights in stochastic portfolio theory. Inspired by volatility stabilized market models introduced by Robert Fernholz and Ioannis Karatzas (2005), we characterize the class of polynomial diffusion models for the asset price process whose market weights process is again a polynomial diffusion process on the unit simplex. Explicit parameter conditions assuring the existence of relative arbitrages with respect to the market portfolio are given and the connection to non-attainment of the boundary is discussed. We also consider extensions to models with jumps and the computation of optimal relative arbitrage strategies.

David Hobson (Imperial College London)

"On the value of being American"

The virtue of an American option is that it can be exercised at any time. This right is particularly valuable when there is model uncertainty. Yet almost all the extensive literature on American options assumes away model uncertainty. We quantify the potential value of this flexibility by identifying the supremum on the price of an American option when no model is imposed on the data, but rather any model is required to be consistent with a family of European call prices. The bound is enforced by a hedging strategy involving these call options which is robust to model error.
Joint work with Anthony Neuberger (Cass).

Rama Cont (Imperial College London)

High frequency trading in limit order markets: From point processes to free-boundary problems

The advent of high frequency trading has changed the landscape of financial markets, leading to a heterogeneous environment where market participants with a wide range of trading frequencies interact through the limit order book. We propose a stochastic model for dynamics of price and order flow in a limit order market, which captures the coexistence of high frequency and low frequency order flow and examines the consequences of this heterogeneity on price dynamics, volatility and liquidity. Based on a detailed empirical study of high frequency order flow in the SP futures market, we argue that the properties of the order flow point to a multi-scale heavy traffic regime, which we contrast with other scaling regimes studied in the recent literature.

We show that in this scaling limit the limit order book may be represented as a measure-valued Markov process whose dynamics may be described by a (stochastic) free boundary problem. Our model provides insights into how the interaction of high and low-frequency trading affects the dynamics of supply, demand and prices in limit order markets.

Mathias Beiglböck (University of Vienna)

"The Geometry of Multi-Marginal Skorokhod Embedding"

During the last 50 years the Skorokhod embedding problem has become an important classical problem in probability theory and a number of solutions with particular optimality properties have been constructed. Recently a unified derivation of many of these solutions has been obtain through a new approach inspired by the theory of optimal transport.

Concerning applications in model-independent finance, the multi-period extension of the Skorokhod embedding problem is of particular interest. Using the classical techniques from stochastic analysis this multi-period version is significantly more difficult – first important results are due to Brown--Hobson--Rogers, Henry-Labordere--Obloj--Spoida—Touzi (multi-marginal Azema-Yor embedding under certain conditions) and more recently Cox--Obloj--Touzi (multi-marginal Root embedding).

Here we show that the transport approach can also be used to extend all the classical optimal solutions to the multi-marginal Skorokhod problem. In particular we establish that these constructions share a common geometric structure. This has further applications to the martingale optimal transport problem.

Ioannis Karatzas (Columbia University New York)

“Lyapounov Functions as Portfolio-Generators”

Back in 1999, E. Robert Fernholz introduced a construction that was both spectacular and remarkably easy to prove. He showed that for a certain class of "functionally-generated" portfolios, it is possible to express the wealth they generate, denominated in terms of the total market capitalization, solely in terms of the individual companies' market weights; and to do so in a pathwise manner, that does not involve stochastic integration. The discovery paved the way for finding simple, structural conditions on large equity markets - that involve more than one stock, and typically thousands - under which it is possible to outperform the market portfolio.

Although well-known, celebrated, and quite easy to prove, Fernholz’s construction has been viewed over the years as somewhat “mysterious". In this talk, and in the work on which the talk is based, we hope to make Fernholz’s results a bit more celebrated and a bit less mysterious, via an interpretation of portfolio-generating functions as Lyapunov functions for the vector process of relative market weights. We then use this approach to settle a question about functionally-generated portfolios that has been open for 10 years.

This is joint work with Prof. Johannes RUF, U.C. London.

Kostas Kardaras (London School of Econmics and Political Science)

“Incomplete stochastic equilibria with exponential utilities close to Pareto optimality”

We study existence and uniqueness of continuous-time stochastic Radner equilibria in an incomplete markets model. An assumption of "smallness" type - imposed through the new notion of "closeness to Pareto optimality" - is shown to be sufficient for existence and uniqueness. Central role in our analysis is played by a fully-coupled nonlinear system of quadratic BSDEs. (Joint work with Hao Xing and Gordan Žitković)

Dörte Kreher (Humboldt-Universität zu Berlin)

“A high frequency limit for a limit order book model with state dependent order dynamics”

In this talk we study a one-sided limit order book (LOB) model, in which the order dynamics depend on both, the current best bid price and the current volume density function. For the joint dynamics of the best bid price and the standing buy volume density we derive a weak law of large numbers, which states that the LOB model converges to a continuous-time limit when the size of an individual order as well as the tick size tend to zero and the order arrival rate tends to infinity. In the scaling limit the LOB dynamics follows a coupled non-linear ODE-PDE system. The talk is based on joint work with U. Horst.

Soumik Pal (University of Washington)

"Exponentially concave functions, optimal transport, and relative arbitrage"

A real-valued function on the set of probability measures on some measurable space is exponentially concave if the exponential of that function is concave. Well-known examples include (multiple of) Shannon's entropy functional for probability measures on Euclidean spaces and the discrete entropy on the unit simplex of probability measures on a finite set. We will connect these functions to (i) a remarkable class of Monge-Kantorovich optimal transport problems, and (ii) building portfolios that are relative arbitrages with respect to the market. Next we show that solutions to a large class of classical transport problems on Euclidean spaces with strictly convex cost functions can be recovered as gradients of exponentially concave functions on probability measures (e.g., the L2 - Wasserstein transport). This is a novel representation that bypasses the usual c-concave functions. The representation is intimately related to the theory of Large Deviations. Using this, we will also describe how one can build new relative arbitrage opportunities with respect to the market portfolio.

Damir Filipovic (Ecolé Polytechnique fédérale de Lausanne)

"On the Relation between Linearity-Generating Processes and Linear-Rational Models"

We review the notion of a linearity-generating (LG) process introduced by Gabaix (2009) and relate LG processes to linear rational (LR) models studied in Filipovic, Larsson, and Trolle (2014). We show that every LG process can be represented as an LR model of the same dimension. More importantly, we identify those (m + 1)-dimensional LG processes that can be represented as m-dimensional LR models. We show that these are the only LG processes that are stationary and mean reverting after exponential scaling. We highlight the ease with which LR models can be specified and be made consistent with nonnegative interest rates.
This is joint work with Martin Larsson and Anders Trolle.

Elise Gourier (Princton University)

"Option pricing in a quadratic variance swap model"

In this talk I present a novel method to price European options in a quadratic variance swap model. The multivariate state process is characterized by a quadratic diffusion function. The variance swap curve is quadratic in the state variable and available in closed form. Our pricing method relies on the polynomial preserving property of quadratic jump-diffusion processes, which allows us to approximate the conditional moments of the log price. The characteristic function is obtained using an Edgeworth expansion, and option prices are recovered with standard Fourier inversion. I illustrate how the proposed option pricing method can be used in a portfolio allocation exercise, where the investor holds a portfolio with variance swaps, the underlying index, bonds and options.

Fred Espen Benth (University of Oslo)

"Modelling energy forward prices"

We introduce random field models coming from stochastic partial differential equations or ambit fields as models for the dynamics of forward prices in energy markets. Various aspects like stochastic

volatility, representation and approximation are analysed. The context of arbitrage relative to the market portfolio within the framework of Stochastic Portfolio Theory.

Beatrice Acciaio (London School of Economics and Political Science)

"Robust Pricing by Informed Investors"

In a model-independent framework, where semi-static trading opportunities are available to all market participants, we study super-hedging prices for agents having access to different information. A crucial role is played by the notion of semi-static completeness, which is the natural extension in this context of the predictable representation property. Under structural assumptions, we find that informed agents compute super-hedging prices using only those probability measures that render the additional information inconsequential. (Joint work with Martin Larsson)

Jan Kallsen (University of Kiel)

"Are American options European after all?"

Christensen (Mathematical Finance 24, 2014, 156-172) has introduced an efficient numerical approach for obtaining upper bounds of American option prices in diffusion models. It relies on approximating the value of the American option by European options with a larger payoff. In this talk we discuss the question whether the value of an American option actually coincides in the continuation region with that of a properly chosen European payoff. Or, in the language of financial engineering: can an American option be hedged statically with European calls/puts of all strikes? In analytical terms this boils down to the question whether a harmonic function solving a free boundary problem can be extended to a harmonic function on the whole space.

Christoph Czichowsky (London School of Economics and Political Science)

"The risk tolerance process and the sensitivity of optimal investment and consumption"

In this talk, we investigate the sensitivity of optimal trading strategies and consumption streams with respect to the current level of wealth. It turns out that both sensitivities can be expressed via the so-called risk tolerance process. They appear quite naturally in a various expansions of portfolio optimisation problems and therefore allow us to asses the "robustness" of the optimisers.

The talk is based on joint work with Jan Kallsen and Johannes Muhle-Karbe.

Walter Schachermayer (University of Vienna)

"Duality Methods in Portfolio Optimization under transaction costs"

We review several recent results for portfolio optimization under proportional transaction costs, such as a Tobin tax. Special emphasis will be put on financial models based on fractional Brownian motion.