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Infinite monster groups

Examples are free Burnside groups, Tarski monsters, Gromov's monster, Thompson's groups. My results include:

— The construction of continuously many non-isomorphic finitely generated groups with strong fixed-point properties. In particular, groups with Kazhdan's property (T) that are simple and have the fixed point property for actions on spaces in $\mathcal{X}_{ac}$ of finite covering dimension that are mod-p acyclic for some prime p [1].
— Details on Gromov's construction of a random group with no coarse embedding into a Hilbert space [2].
Research directions are the study of *analytic properties of infinite monster groups* and construction of new monsters (e.g. a *non-sofic group*).

Random groups

There are two distinct aspects of infinite random groups. The first one is the study of a typical (or generic) representative of a given class of groups. This has already various combinatorial, statistical, and topological interpretations. The second aspect is based on the Erdös probabilistic method and provides some exotic random groups (e.g. Gromov's random group coarsely containing an expander). My results include (see also [2]):

— A generic finitely presented group is surface-like [3,4];
— The strengthened Hanna Neumann conjecture holds for every H and a generic K (two finitely generated subgroups of a free group) [5].
Research directions are *new models of random groups and algebras*, strong fixed point properties of random groups.

Analytic properties of groups

An analytic group property is one which depends on the harmonic analysis or the unitary representation theory of the group. Historically foundational examples are the amenability and Kazhdan's property (T). Recent offsprings are Gromov's a-T-menability (=the Haagerup property), coarse amenability, and coarse embeddability into a Hilbert space. My results include:

— The C^*-simplicity of relatively hyperbolic groups [6];
— The first examples of bounded geometry metric spaces which are not coarsely amenable but admit a coarse embedding into a Hilbert space [7];
— The Haagerup property and the property of Rapid Decay of infinitely presented small cancellation groups [8,9].
Research directions are the existence of a coarsely embeddable but not coarsely amenable *group* (a big open problem in operator algebras) and the C^*-simplicity of infinite monsters.

Sofic and Hyperlinear groups

A group is sofic if its elements can be approximated in a certain sense by permutations in the Hamming distance. This notion was introduced by Gromov (1999). Gottschalk's surjuctivity conjecture (1973) in topological dynamics, Kaplansky's direct finiteness conjecture (1956) in the theory of rings, and Alain Connes' embedding conjecture (1976) in operator algebras have recently been established affirmatively for all sofic groups. My results include:

— The study of linear sofic groups and algebras [10], and of asymptotic approximations of infinite groups [11].
Research directions are questions on the *surjunctivity and hyperlinearity* of linear sofic groups.

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[11] G.N. Arzhantseva, *Asymptotic approximations of finitely generated groups*, submitted, (2012).