Department of Pure Mathematics

Module 110PMA207 – Linear Algebra

Assignment 9

---

1. Let $x = (2, -1, 3) \in \mathbb{R}^3$ and let $y^* : \mathbb{R}^3 \to \mathbb{R}$ be a linear functional defined by $y^*(e_1) = -1$, $y^*(e_2) = 3$, $y^*(e_3) = 2$.

   (a) Compute $x^{**}(y^*)$.

   (b) Find a non-zero functional $y_0^* : \mathbb{R}^3 \to \mathbb{R}$ such that $x^{**}(y_0^*) = 0$.

2. Let $\varphi : \mathbb{R}^5 \to \mathbb{R}^3$ be a linear mapping and let

$$A = \begin{pmatrix} -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 \end{pmatrix}$$

   be the corresponding matrix with respect to the natural bases in $\mathbb{R}^5$ and $\mathbb{R}^3$ respectively. Further let φ^* be the dual mapping of φ and let $f : \mathbb{R}^3 \to \mathbb{R}$ be a linear functional defined by $f(e_1) = -2$, $f(e_2) = 5$, $f(e_3) = 0$.

   (a) Compute $\big(\varphi^*(f)\big)\big((-1, 1, -1, 1, -1)\big)$.

   (b) Find a non-zero vector $x \in \mathbb{R}^5$ such that $\big(\varphi^*(f)\big)(x) = 0$.

   (c) For which vectors $x \in \mathbb{R}^5$ we have $\big(\varphi^*(f)\big)(x) = 0\,$?

3. Compute the determinant of the following matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 4 & -8 \\ 1 & 3 & 9 & 27 \\ 1 & -4 & 16 & -64 \end{pmatrix}$$

4. Write the determinant of the matrix

$$M = \begin{pmatrix} 5-\lambda & -2\frac{2}{5} & 4 \\ 5 & -1-\lambda & 0 \\ 0 & 1\frac{2}{5} & -4-\lambda \end{pmatrix}$$

   as a polynomial in λ.