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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 3

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11. Let $S = \{p, q, r, s\}$ be a set and let "$\circ$" be the operation on S defined by the following multiplication table:

$\circ$	p	q	r	s
p	r	s	p	q
q	s	r	q	p
r	p	q	r	s
s	r	p	s	r

(a) Find the neutral element of S and show that every element has an inverse.

(b) Show that the operation "$\circ$" is not commutative.

(c) Is the operation "$\circ$" associative?

12. Let $(G, *_G)$ and $(H, *_H)$ be groups and let the operation "$\circ$" on $G \times H$ be defined as follows:

$$\langle g_1, h_1\rangle \circ \langle g_2, h_2\rangle := \langle g_1 *_G g_2, h_1 *_H h_2\rangle$$

Show that $(G \times H, \circ)$ is a group and that it is abelian iff G and H are both abelian.

13. Show that $C_2 \times C_3 \simeq C_6$.
Hint: Let $C_2 = \{e, a\}$ and $C_3 = \{e', b, b^2\}$, and consider $c, c^2, c^3, \ldots$, where $c = \langle a, b\rangle$.

14. (a) Show that for any positive integers p and q with $\gcd(p, q) = 1$ we have $C_p \times C_q \simeq C_{pq}$.

(b) Show that $C_3 \times C_3$ is not isomorphic to C_9.

15. Let T, C, O, D and I donote the groups of rigid motions of the five Platonic solids, called *tetrahedron, cube, octahedron, dodecahedron*, and *icosahedron*.
Compute $|T|$, $|C|$, $|O|$, $|D|$ and $|I|$.