

## Seminar Topics

In most of the cases in which multiple sources are listed with a ”+” between them there is large redundancy, so they’re not necessarily the longest topics to prepare. A more detailed plan is for most of the weeks to be discussed with the corresponding students. All the books and papers listed in the references are available for free in PDF format through the ETH network.

- Week 0 (20.2.) Introduction
- Week 1 (27.2.) Equivalence between AC and WOP ([2]: 60-65 + [3]: 151)
  - Week 2 (5.3.) Equivalent forms or consequences of CH (from [5]: Chapter 16)
- Week 3 (12.3.) Equivalent forms or consequences of CH (from [5]: Chapter 16)
- Week 4 (19.3.) Banach-Tarski and Hausdorff Paradox ([3]: 115-127 + [2]: 177-181)
- Week 5 (26.3.) Robinson’s Decomposition ([2]: 181-187)
  - Week 6 (9.4.) Riddles using the Axiom of Choice ([3]: 110-114 + provided material)
- Week 7 (16.4.) Equivalent forms of AC ([2]: Thm. 6.3, Thm. 6.4)
- Week 8 (23.4.) Equivalence between the Ultrafilter Theorem and  $P_3$  ([2]: Lemma 6.9, Thm. 6.10)
- Week 9 (30.4.) CH implies the existence of Ramsey UF ([2]: Thm. 4.1, Prop. 4.2, Prop. 11.9)
- Week 10 (7.5.) CH implies the existence of non-Ramsey P-points ([2]: Prop 14.9)
- Week 11 (14.5.) CH implies the existence of Magic Sets ([1]: Thm 8.1-8.5 + [4]: Lemma 3.4 + provided material)
- Week 12 (21.5.) Axiom of Determinacy: definition, inconsistency with AC, implication that all subsets of  $\mathbb{R}$  are Lebesgue measurable. ([3]: Thm 15.25 + provided material)
- Week 13 (28.5.) Banach–Kuratowski matrices ([2]: Suite, Chapter 29)

## References

- [1] A. Berarducci, D. Dikranjan, **Uniformly approachable functions and spaces**, Rendiconti dell’Istituto di Matematica dell’Universita di Trieste, an International Journal of Mathematics, 25, pages 23-53 (1993).
- [2] L. Halbeisen, **Combinatorial Set Theory: With a Gentle Introduction to Forcing**, (2nd. ed.), [Springer Monographs in Mathematics], Springer-Verlag, London (2017).

- [3] L. Halbeisen, R. Krapf, **Eine Entdeckungsreise in die Welt des Unendlichen**, Springer Spektrum Berlin, Heidelberg (2023).
- [4] L. Halbeisen, M. Lischka, S. Schumacher, **Magic Sets**, Real Anal. Exchange 43 (1) 187 - 204 (2018).
- [5] P. Komjáth, V. Totik, **Problems and Theorems in Classical Set Theory**, [Problem Books in Mathematics], Springer-Verlag, New York (2006).