OPEN QUESTION IV: THE CLIFFORD AND CHEKANOV TORI ARE ISOTOPIC (THROUGH IMMERSIONS)

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There is an h-principle for Lagrangian immersions into $\mathbb{C}^n$ with specified symplectic area class (see [3]). In particular, if two monotone tori with the same monotonicity constant are isotopic through Lagrangian immersions, they are isotopic through monotone Lagrangian immersions with the same monotonicity constant. This tells us that there exists such an isotopy S connecting the Clifford and Chekanov tori in $\mathbb{C}^2$.

It is known that these two tori are not Hamiltonian isotopic [2] and that an isotopy through monotone Lagrangian embeddings with the same monotonicity constant is a Hamiltonian isotopy. Therefore the isotopy S must at some intermediate stage pass through immersed Lagrangian monotone tori.

Question 1. *Construct such an isotopy explicitly. How many "Reidemeister moves" do you need to get from one to the other?*

By Reidemeister move, I simply mean how many times must the Lagrangian pass through itself for an optimal generic immersed isotopy? Note that these may need to be dangerous self-tangencies (where the Legendrian lift[1] in $\mathbb{C}^n \times S^1$ is itself immersed). Indeed I suspect that such dangerous self-tangencies are necessary (that the Chekanov disc-counting invariant which distinguishes the Hamiltonian isotopy classes is somehow derived from the Legendrian contact homology of the lift).

It may or may not be helpful to contemplate the following picture of the two tori ([1], Section 5). Let $(x, y) \mapsto xy$ be the conic Lefschetz fibration on $\mathbb{C}^2$ with one singular fibre (so the fibres are copies of T^*S^1). The Clifford torus sits over the unit circle and intersects each fibre in the zero section of T^*S^1. The Chekanov torus sits over another circle which does not encircle the origin.

REFERENCES

[1] D. Auroux, 'Mirror symmetry and T-duality in the complement of an anticanonical divisor' J. Gökova Geom. Topol. 1 (2007), 51–91

[2] Y. Chekanov, 'Lagrangian tori in a symplectic vector space and global symplectomorphisms', Math. Zeit. Volume 223, Number 1 (1996) 547–559

[3] J. D. Evans and J. Kędra, 'Remarks on monotone Lagrangians in $\mathbb{C}^n$', 2011 preprint arXiv:1110.0927

[1] The h-principle for monotone Lagrangians is proved by first lifting to a Legendrian in $\mathbb{C}^n \times S^1$ and then applying the h-principle for Legendrian immersions there. The lift is constructed by finding a circle-valued primitive for the pullback of the Liouville form.

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