Noname manuscript No.
(will be inserted by the editor)

Modeling Operational Risk Depending on Covariates. An Empirical Investigation.

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Received: date / Accepted: date

Abstract The importance of operational risk management in financial and commodity markets has significantly increased over the last decades. This paper demonstrates the application of a non-homogeneous Poisson model and dynamic Extreme Value Theory (EVT) incorporating covariates on estimating frequency, severity and risk measures for operational risk. Compared with a classical EVT approach, the dynamic EVT has a better performance with respect to the statistical fit and realism. It also has good flexibility to different types of empirical data. In our model we include firm-specific covariates associated with Internal Control Weaknesses (ICWs) and show empirically that firms with higher incidences of selected ICWs have higher time-varying severities for operational risk. Our methodology provides risk managers and regulators with a tool that uncovers the non-obvious patterns hidden in operational risk data.

Keywords Covariates · Dynamic EVT · Generalized Additive Models · Internal Control Weaknesses · Operational Risk · Risk Measures · Value-at-Risk

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1 Introduction

Based on incomparability and lack of reliability, the new Basel documents (Basel Committee on Banking Supervision, 2016, 2017) herald the end of the Advanced Measurement Approach (AMA) for the calculation of regulatory capital towards operational risk. As a result, the future of the Loss Distribution Approach (LDA) based methods for operational risk becomes ambiguous. Nevertheless, operational risk losses continue to pose a major threat to large financial institutions. The first author of this paper (P.E.) is quoted on RISK.net (March 13, 2013) with the words "In the past three years we have seen, again and again, massive legal claims against banks that dwarf the sum of all the other operational risk events. That's a major issue, and I don't think that many of the current risk models are reflecting this reality." The Economist of August 18, 2016, reports that legal settlements for operational risk losses since 2009 amount to 219 billion USD with Bank of America topping the list with 70 billion USD (50% of its market capitalization). To address some of the underlying issues, in this paper, we apply a dynamic extreme value theory (EVT) model based on a non-homogeneous Poisson process incorporating covariates to estimate frequency, severity and risk measures for operational risk. Based on this method, scenario tests are easy to implement and useful empirical analyses can be obtained. As a consequence, the LDA methods can still be meaningful for internal use by a bank, an insurance company or by research institutions interested in the analysis of industry-wide operational risk losses.

The purpose of this paper is threefold: first, a new dataset with adequate sample size is used, which makes comprehensive analysis possible; second, it yields an empirical assessment of the performance of the new dynamic model developed by Chavez-Demoulin et al. (2016), which is a flexible statistical approach for the modeling of operational risk as a function of covariates; third, our paper provides a first analysis statistically linking risk measures for operational risk to covariates based on internal control weaknesses (ICWs). The covariates considered in this paper are related to the firm size, financial health, reporting complexity and a dummy covariate for the financial crisis.

The rest of this paper is organized as follows. Section 2 provides a brief review of the relevant literature. Section 3 introduces the dynamic EVT model implemented in this paper. Section 4 describes the operational risk dataset used throughout this paper. In Section 5, we compare and contrast the performances of classical and dynamic EVT models for Financial Services and Manufacturing. We also show how a dynamic model yields a more realistic estimation for underlying risk measures. Section 6 contains the main contribution of the paper: based on Chavez-Demoulin et al. (2016), we develop a dynamic operational risk model for five different industries based on seven covariate factors, referred to as ICWs. Section 7 provides conclusions. We also discuss limitations to the method provided as well as an outlook for further study.

2 Literature Review

The Basel Committee has decided to replace the AMA approach for the calculation of regulatory capital for operational risk by a standardized formula based on so-called Business Indicators; see Basel Committee on Banking Supervision (2017).

The motivation behind the document mainly relates to the failure of industry to come up with a sufficiently robust and widely applicable statistical methodology towards regulatory capital calculations (see for instance Basel Committee on Banking Supervision 2014 and Basel Committee on Banking Supervision 2017). This possible shortcoming was early on voiced in the academic literature (e.g. Danielsson et al. 2001) and was also very much understood by the insurance regulators; see the relevant documents under the Swiss Solvency Test (SST, FINMA) as well as the EU guidelines for Solvency II. In the wake of the 2007-2009 financial crisis (referred to throughout our paper as the financial crisis), in general, the regulatory pendulum swayed away from the use of internal models; see also Embrechts (2017). In the case of operational risk, a detailed discussion can be found in Peters et al. (2016). Because of the considerable (quality control) relevance of operational risk, financial institutions ought to be encouraged to internally analyze operational loss data bases and compare and contrast the findings with industry-wide practice. It is precisely at this point that the methodology used in this paper may be useful.

The modeling of operational risk is focused on both the frequency as well as severity of loss events, resulting in statistical models for the loss distribution. It is common for banks to use Poisson or negative binomial processes for frequency estimation. Regarding the loss distribution, surveys from the Basel Committee on Banking Supervision (Basel Committee on Banking Supervision 2009*a* and Basel Committee on Banking Supervision 2009*b*) showed that almost all banks modeled the body and tail separately. To model the body of the loss distribution, the empirical and lognormal distribution are widely applied. For the tail (severity) estimation, the generalized Pareto distribution (GPD) is frequently used. Institutions face many modeling choices when they attempt to measure operational risk, and using different models for the same institution can lead to materially different risk estimates (e.g. Dutta and Perry 2007 as well as the above 2014 BCBS document). The former paper also provides some general modeling guidelines; see our Section 7. For a comprehensive overview and detailed discussion of the models used, we refer to Cruz et al. (2015).

Concerning the tail area, EVT has been widely applied in finance and insurance as thoroughly described in Embrechts et al. (1997) and McNeil et al. (2015). Mizgier et al. (2015) applied EVT to data from financial services and manufacturing industry sectors. They propose to use tools from the field of operations management such as process improvement and combine them with capital adequacy to manage operational risk. At the same time, a naive use of EVT-type models may disregard important features, like regime changes often present in financial data; for an early paper raising this concern, see Diebold et al. (2001). The dynamic EVT model proposed in Chavez-Demoulin et al. (2016) combines the obvious relevance of EVT modeling but at the same time tries to address some of the criticisms (for a related paper, see Hambuckers et al. 2018).

With respect to the choice of risk measure, for operational risk the Basel II guidelines originally stipulated for internal modeling purposes a 99.9%, 1-year Value-at-Risk (i.e., a “one in 1,000 year event”). An alternative risk measure considered is Conditional Value-at-Risk (also termed Expected Shortfall, Rockafellar and Uryasev 2000). For a multi-period extension of these two risk measures in operational risk context we refer to Mizgier and Wimmer (2018); we will come back to the use of these risk measures for operational risk management in the subsequent sections.

Several studies have been conducted on the relation between operational risk and firm-specific covariates, such as firm size (Shih et al. 2000; Ganegoda and Evans 2013), gross revenue (Na et al. 2006), corporate governance and CEO incentives (Chernobai et al. 2011). Motivated by the accounting literature on the determinants of ICWs (Doyle et al. 2007), a comprehensive analysis of firm-specific and macroeconomic variables associated with ICWs that contribute to the frequency of operational risk events has been provided by Chernobai et al. (2011). However, little of the literature explains the effects of these covariates on the severity of operational risk.

3 A Dynamic EVT Approach

EVT is a branch of statistics concerned with limiting laws for extreme values in large samples. There are two main types of statistical EVT models: the Block Maxima approach models the largest observations from a set of blocks dividing a large sample; and the peaks-over-threshold (POT) approach models the exceedances over a high threshold. The POT approach uses the data of extreme values more efficiently because it incorporates all the large observations exceeding the threshold. Classical EVT holds under the i.i.d. or weak dependence assumptions (Embrechts et al. 1997). For the purpose of this paper we concentrate on the dynamic EVT-POT model developed by Chavez-Demoulin et al. (2016) which is very much based on the ideas outlined in Chavez-Demoulin and Davison (2005). In Chavez-Demoulin et al. (2016), loss frequency is modelled through a non-stationary Poisson process with intensity λ, whereas the loss severities follow EVT-type parametric models based on the generalized Pareto distribution, GPD(ξ, β), with shape parameter ξ and scale parameter β. In both cases (frequency, severity), the parameter vector involved, $\boldsymbol{\theta} = (\lambda, \xi, \beta)$, may vary dynamically and can be a function of several underlying covariates.

In the dynamic POT method, the number of exceedances is assumed to follow a non-homogeneous Poisson process with the following intensity function:

$$\lambda(x,t) = exp(f_\lambda(x) + h_\lambda(t)), \tag{1}$$

where x denotes covariates for which sufficient data is available, t corresponds to time allowing for non-stationarity and the real-valued functions f_λ and h_λ are smoothing functions. This type of model is common in the realm of generalized additive models (GAM); see Wood (2006) and Taylan et al. (2007).

Regarding the approximating GPD(ξ, β) for the excess distribution, we also assume that the parameters ξ and β may depend on time and certain covariates and have a similar form as (1), given by

$$\xi(x,t) = f_\xi(x) + h_\xi(t), \tag{2}$$
$$\nu(x,t) = f_\nu(x) + h_\nu(t), \tag{3}$$

for some real-valued smoothing functions f and h. In (3), the parameter β is reparameterized as

$$\nu = log((1+\xi)\beta), \tag{4}$$

for $\xi > -1$, to make it orthogonal to ξ with respect to the Fisher information metric (Chavez-Demoulin 1999; Chavez-Demoulin et al. 2016). Unlike (1), the fitting of (2) and (3) cannot be solved by a standard GAM procedure. In Chavez-Demoulin et al. (2016) a backfitting algorithm together with a blackend bootstrap procedure for the construction of confidence intervals is discussed.

Given the observations $\boldsymbol{z}_i = (\boldsymbol{t}_i, \boldsymbol{x}_i, \boldsymbol{y}_i), i \in \{1, .., N_u\}$, in which t_i represents time, x_i represents the selected covariate, y_i represents the excess amount over a given threshold u, and N_u is the number of exceedances, the resulting penalized log-likelihood function is given by:

$$l^p(f_\xi, h_\xi, f_n u, h_n u; \boldsymbol{z}) = l^r(\boldsymbol{\xi}, \boldsymbol{\nu}; \boldsymbol{y}) - \gamma_\xi \int_0^T h_\xi''(t)^2 dt - \gamma_\nu \int_0^T h_\nu''(t)^2 dt, \tag{5}$$

where

$$l^r(\boldsymbol{\xi}, \boldsymbol{\nu}; \boldsymbol{y}) = \sum_{i=1}^{N_u} l^r(\boldsymbol{\xi}_i, \boldsymbol{\nu}_i; \boldsymbol{y}_i), \tag{6}$$

for

$$l^r(\boldsymbol{\xi}_i, \boldsymbol{\nu}_i; \boldsymbol{y}_i) = l(\boldsymbol{\xi}_i, exp(\boldsymbol{\nu}_i)/(1 + \boldsymbol{\xi}_i); \boldsymbol{y}_i). \tag{7}$$

Here, $\gamma_\cdot \geq 0$ denotes smoothing parameters, in which larger values lead to smoother fitted curves.

By maximizing the penalized log-likelihood function (5) with the backfitting algorithm, the estimated (possibly dynamic) parameter $\hat{\boldsymbol{\theta}} = (\hat{\lambda}, \hat{\xi}, \hat{\beta})$ can be obtained. Based on $\hat{\boldsymbol{\theta}}$, the risk measures VaR_α and ES_α given a fixed covariate x and time t can be estimated by

$$\widehat{VaR}_\alpha = u + \frac{\hat{\beta}}{\hat{\xi}} \left(\left(\frac{n'}{\hat{\lambda}} (1 - \alpha) \right)^{-\hat{\xi}} - 1 \right), \tag{8}$$

$$\widehat{ES}_\alpha = \begin{cases} \frac{\widehat{VaR}_\alpha + \hat{\beta} - \hat{\xi} u}{1 - \hat{\xi}}, & \hat{\xi} \in (0, 1), \\ \infty, & \hat{\xi} \geq 1, \end{cases} \tag{9}$$

where n' is the number of all losses given a certain covariate x and time t (see McNeil et al. 2015 (p. 155) and Chavez-Demoulin et al. 2016 for further details on this procedure). In (8) and (9) we restrict the formulation to the case of a positive shape parameter ξ; this indeed corresponds to the often observed power tail behavior of operational risk loss distributions. The cases $\xi \leq 0$ however can also be dealt with within the same framework. By now, EVT methodology has numerous applications throughout science, technology and economics. The textbook references above contain several examples, mainly to insurance and finance. One of the early applications to finance is Longin (1996). For applications of dynamic EVT models and covariate modeling, see for instance Pauli and Coles (2001), Chavez-Demoulin and Davison (2005), Eastoe and Tawn (2009) and Yee and Stephenson (2007). If we were asked to single out one application showing the fundamental importance of EVT towards risk-type applications, then this has to be the sinking of the M. V. Derbyshire as presented in Heffernan and Tawn (2001) and Heffernan and Tawn (2003). For a brief account of the resulting court case, see Heffernan and Tawn (2004).

4 The Data

Data quality as well as availability are major concerns in operational risk research. A full statistical analysis typically combines internal, external and expert opinion data. The combination of such different data sources poses some non-trivial problems as highlighted early on by Baud et al. (2002), Baud et al. (2003) and Cope et al. (2009). For a recent textbook reference, see Chapter 15 of Cruz et al. (2015). One of the main data sources for internal data is the Loss Data Collection Exercise (LDCE) implemented by the Basel Committee. The latest 2008 LDCE (Basel Committee on Banking Supervision 2009*b*) collected 10.6 million internal losses for at least 3 years from 121 banks in 17 countries. An important industry based initiative is the Operational Riskdata eXchange Association (ORX) which has, since 2002, collected operational loss events from 89 members. By the end of 2015, the Global Banking Database in ORX contained 490,044 events.

The full data set underlying this paper comprises 32,566 operational loss events across 21 industries over the period 1971 to 2015 and was provided by the SAS Institute, Switzerland. For each event, information such as firm name, event description, loss value, business line, event type, geographical region of both legal entity and incident, beginning and ending year of event, and settlement time are given. Some of the events include accounting information such as revenue, asset value, equity value, the number of employees, net income, and liability. Compared with the other providers, the SAS-data covers industries beyond the banking sector and also provides useful accounting information relevant for the analysis of the impact of ICWs as covariates. The data collection time period covers the financial crisis. In line with Mizgier et al. (2015), in our analysis we concentrate on Financial Services (FS) and Manufacturing (M), and use the newly available extended data set resulting in 20,648 losses; see Table 1 for details.

5 Statistical Analyses for Financial Services and Manufacturing

5.1 Preliminary Comments

In this section we gradually demonstrate the versatility of the dynamic EVT approach as developed in Chavez-Demoulin et. al (2016). The need for such an approach becomes clear from Figure 1 depicting loss frequency as well as loss severity for both FS and M.

The data for both industries exhibit possible non-stationarity. For instance the frequency for FS shows a peak around the financial crisis, whereas for M a less pronounced but broader peak appears. For loss severity, there is clear qualitative difference between FS and M; in the case of M, loss severity peaks rather spiky around 2000 whereas for FS, loss severities show an increasing trend over the period 1990 - 2008. In a first analysis we will model the data for FS and M through Classical EVT-POT; later we will refine our analyses modeling possible non-stationarity in both frequency as well as severity. We will also show which influence such a more dynamic approach has on reported risk measures. In Section 6 we then consider dynamic models based on further covariates and in particular concentrate on the influence of ICWs. Whereas we will introduce the most relevant notation and results from EVT in order for the reader to fully understand our

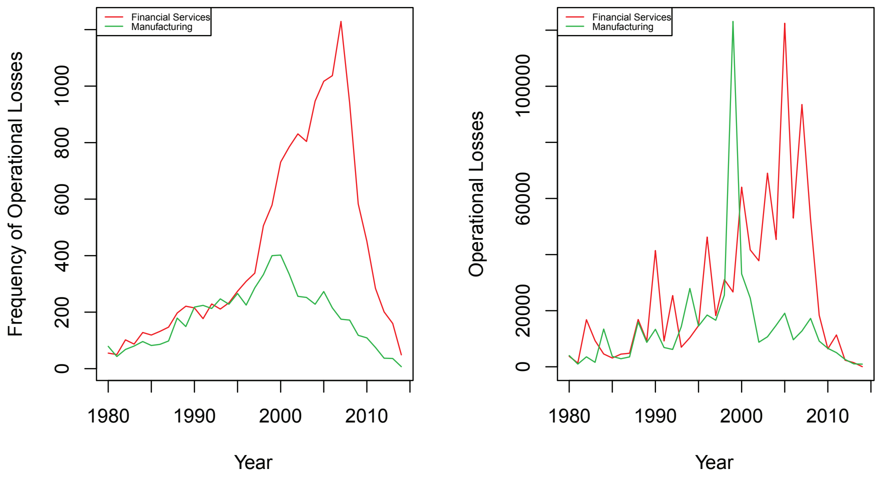


Fig. 1: Time series of yearly frequencies and yearly severities of operational risk losses for FS and M ($M).

analyses and conclusions, for full technical details underlying the methodology used we refer the reader to Chavez-Demoulin et al. (2016) and the references therein.

Within the POT approach to EVT, a standard goodness-of-fit test checks whether the fitted residuals $r_i = -log(1 - G_{\hat{\xi}_i,\hat{\beta}_i}(y_i)), i \in \{1, ..., N_u\}$, behave like iid, standard exponential data. The outcome of a POT analysis is always a function of the threshold u used above which the GPD asymptotics are implemented. We therefore perform a threshold-sensitivity analysis over a wide range of u-values (0.5 till 0.9 quantile) as proposed for instance by de Fontnouvelle et al. (2004). Goodness-of-fit will be assessed through Q-Q plots of the r-residuals. For these initial analyses, we use the lowest threshold for which a good fit is obtained. More refined methods for an optimal threshold choice can be implemented, they do however not change the conclusions reached. For the dynamic case, involving covariates, model selection will be based on AIC and Likelihood Ratio Testing (LRT).

5.2 A First EVT-POT Analysis for FS and M

A standard EVT-POT analysis for loss severities for FS and M delivers the following results:

- For FS: u=0.92 quantile (corresponding to a threshold value of $M 97.5) with n=1149, $\hat{\xi} = 0.844$ and $\hat{\beta} = 181$.
- For M: u=0.9 quantile (corresponding to a threshold value of $M 109) with n=628, $\hat{\xi} = 0.723$ and $\hat{\beta} = 166$.

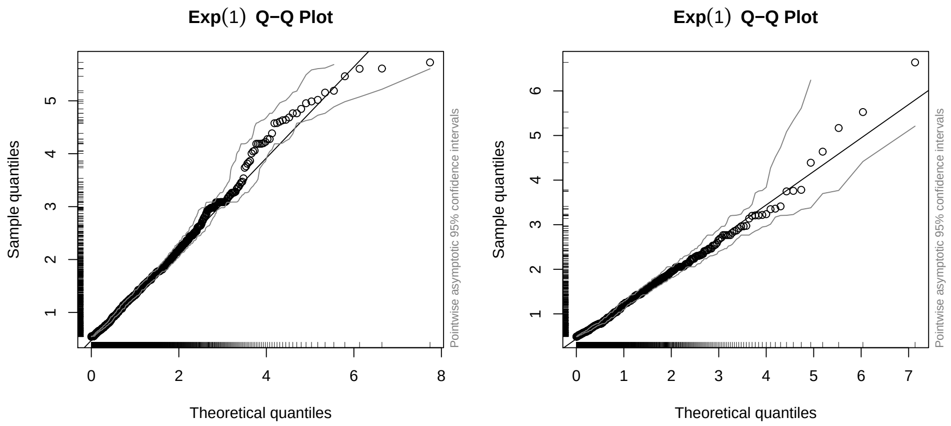


Fig. 2: Q-Q plots for Standard EVT-POT model residuals based on a 0.92 -quantile threshold for FS (left) and M with a 0.9-quantile (right).

Figure 2 contains the goodness-of-fit Q-Q plots for the above models. Both models show heavy-tailedness, i.e., infinite variance, finite mean. This is very much in line with numerous analyses of operational risk data. Note that for the ξ-parametrization of EVT, a value $\xi > 1$ corresponds to an infinite mean model. For $\xi < 0.5$, a finite variance results. Whereas for this analysis we do not use all information available, note already the so-called subexponential character of operational risk, i.e., the largest loss dominates the total sum. For a mathematical discussion on subexponentiality, see Embrechts et al. (1997), Section 8.2.

Still at this more explorative phase of the data modeling for FS and M, we now allow the shape (ξ) and scale (β or ν) parameters to depend on time t as a covariate. Again, based on Q-Q plotting, the following models result:

- For FS: u=0.9 ($M 70) with n=1436, $\hat{\xi}(t) = \hat{c}_\xi$ and $\hat{\nu}(t) = \hat{c}_\nu t$.
- For M: u=0.8 ($M 42) with n=1258, $\hat{\xi}(t) = \hat{h}_\xi^{(3)}(t)$ and $\hat{\nu}(t) = \hat{c}_\nu$.

The Q-Q plots for these best fitting models are given in Figure 3. Throughout the paper, we denote $\hat{h}_\cdot^{(Df)}$ an estimated natural cubic spline, $\hat{c}_\cdot$ denotes an estimated generic constant. The above Df (Degrees of freedom) is estimated through an AIC-based scree-diagram (not shown here). We note that the all-important shape parameter ξ for FS remains a constant, though with a value slightly larger than the value 0.844 for the static case. For M the static value 0.723 for ξ now becomes time dependent; this happens only rarely, i.e often estimated ξ-values are rather robust with respect to time variation in the underlying data.

Both standard and dynamic EVT models yield a good fit given the selected threshold. For both industries, the dynamic models attain the good statistical fits at lower quantile thresholds, which means that the excess distributions in the dynamic models converge to their GDP limits more quickly yielding superior modeling.

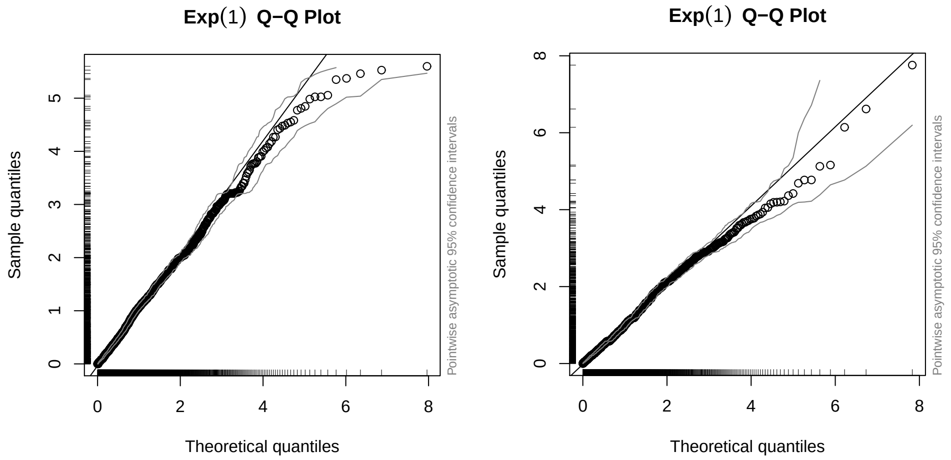


Fig. 3: Q-Q plots for Dynamic EVT-POT based severity models for FS at a threshold 0.9-quantile (left) and 0.8 for M (right).

5.3 EVT-based VaR Estimation for FS

In this section, we estimate the one-year 99.9% Value-at-Risk (VaR) risk measures for a fully dynamic EVT-POT model. We compare and contrast these values with those obtained from a standard EVT-POT approach as presented in Section 5.2. The main reason why we restrict to FS in this section is that for banks the calculation of internationally agreed upon capital requirements is mandatory.

Given the threshold u decided in Section 5.2, dynamic models for λ, ξ and ν (equivalently β) are fitted. The requested VaR can then be calculated from (8). Due to the extreme heavy-tailedness of the data (ξ close to 1) we refrain from reporting ES-values.

For the frequency of operational risk losses for FS we fitted several models, including constant, linear as well as natural cubic splines depending on a number of degrees of freedom. The best fitting model was decided upon through a scree-diagram based on AIC (figure not shown). The resulting best fitting model obtained is

$$log\hat{\lambda}(t) = \hat{h}_{\lambda}^{(7)}(t), \tag{10}$$

i.e., a seven-degree natural spline function. We similarly fitted several dynamic models for the GPD parameters (ξ, β) or (ξ, ν) in case of dynamic parameters for loss values in excess of a sufficiently high threshold (chosen as explained before). Model comparison was based on AIC and LRT resulting in the following model

$$\hat{\xi}(t) = \hat{c}_{\xi}, \qquad \hat{\nu}(t) = \hat{c}_{\nu}t. \tag{11}$$

With the dynamic models decided, we get the time-varying estimated parameter values for both classical and dynamic POT models as shown in Figures 4 and 5.

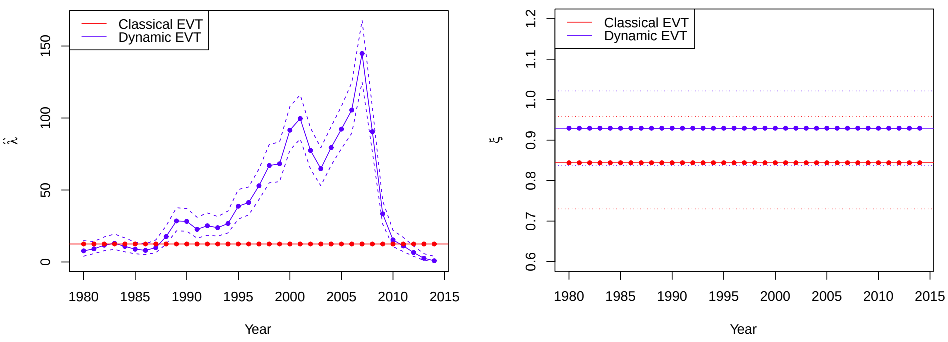


Fig. 4: A comparison of Standard EVT-POT and Dynamic EVT-POT estimates: $\hat{\lambda}$ (left), $\hat{\xi}$ (right) for FS.

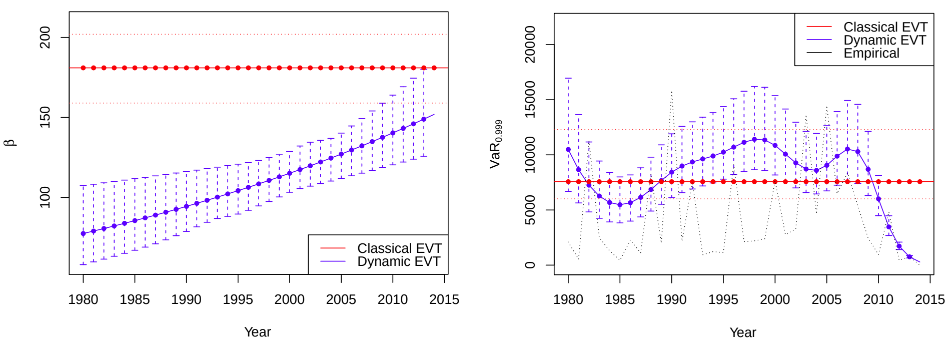


Fig. 5: A comparison of Standard EVT-POT and Dynamic EVT-POT estimates: $\hat{\beta}$ (left), $\widehat{VaR}_{0.999}$ (right) for FS.

Figure 4 (left) highlights the more realistic model for a time-dependent frequency λ, peaking during the financial crisis. Figure 4 (right) for both models stresses the considerable heavy-tailedness of the fitted GPD distributions. As already noted before, the estimated shape parameter ξ for the dynamic model is larger than the static one with the critical value 1 (infinite mean) contained in the 95% confidence interval.

In Figure 5 (right) we have plotted the yearly 99.9% VaR over the modeling period 1980-2015, and this for both models. We also added the empirical loss values. Note that, especially through the latter part of the above period, the dynamic model tracks the empirical values much better. For this period, the static approach considerably overestimates the VaR.

6 The Influence of Industry Type and ICWs on Operational Risk

6.1 Preliminary Comments

In this section we highlight the full power of the dynamic EVT-POT approach as developed in Chavez-Demoulin et al. (2016). In particular, the method is applied to operational risk losses from five industries: Financial Services, Manufacturing, Mining, Utilities, and Information with the following event types:

BDSF Business Disruption and System Failures,
CPBP Clients, Products & Business Practices,
DPA Damage to Physical Assets,
EPWS Employment Practices and Workplace Safety,
EDPM Execution, Delivery & Process Management,
EF External Fraud,
IF Internal Fraud.

Table 1 shows descriptive statistics (sample size, mean, median, summation, standard deviation, minimum, maximum, skewness and kurtosis) of each event type for the five industries displaying different statistical properties.

For most of the industries BDSF is the rarest event whereas it ranks the third most frequent event for Utilities. CPBP dominates for all industries, except possibly for Utilities. The second most frequent event reflects the business characteristics for each industry: DPA has the second highest frequency for Manufacturing, Mining, and Utilities, all of which invest large amount in equipments and infrastructure; EPWS is the second most frequent for Information, strongly personnel driven for customer service and R&D; EF and IF have almost the same frequencies and the overall frequency for fraud is even larger than CPBP, which makes sense because financial services are particularly vulnerable to fraud. The large values of maxima and higher moments of losses (i.e., variance, skewness and kurtosis) all imply relatively high probabilities for extremely large losses; see also our discussion in Section 1.

Tables 2 and 3 show the thresholds confirmed by the sensitivity analysis method mentioned in Section 5.2, the number of exceedances of the relevant threshold and covariates confirmed for frequency and severity models. The cross mark "$\times$" in a column implies that the model is significantly different from the previous one while the tick mark "$\surd$" implies that the difference is not significant. The ticks in the Selection column point to the selected models. So for instance for Financial Services, from Table 2, the optimal model found is $log\lambda = x + h_{\lambda}^{(7)}(t, by\ x)$ [1], $\xi = f_{\xi}(x)$ and $\nu = c_{\nu}t$, whereas for Manufacturing the following model is chosen: $log\lambda = x + h_{\lambda}^{(7)}(t, by\ x)$, $\xi = f_{\xi}(x) + c_{\xi}t$ and $\nu = c_{\nu}$. Note that the variable "x" corresponds to the event type covariates BDSF, CPBP, DPA, EPWS, EDPM, EF and IF.

[1] $h_{\lambda}^{(7)}(t, by\ x)$ here means 7-degree-of-freedom natural spline function of time t and the event type covariate x

Table 1: Descriptive Statistics of operational risk losses (period 1971 - 2015) for Financial Services, Manufacturing, Mining, Utilities and Information (From Top to Bottom) for different Event Types. Loss-values are given in $\$M$.

Event	Size	Sum.	Max	Mean	S.D.	Skew.	Kurt.
Overall	14,359	923,903.34	23,240	64.34	551.21	24.53	765.43
BDSF	77	2,960.04	461.1	38.44	79.84	3.32	15.17
CPBP	6,260	709,386.24	23,240	113.32	784.29	18.21	411.8
DPA	110	5,321.43	1,220.29	48.38	150.99	5.52	38.4
EPWS	342	14,871.55	6,500	43.48	362.49	16.77	296.61
EDPM	699	12,604.12	2,850	18.03	123.1	18.36	406.06
EF	3,439	45,854.40	6,006	13.33	122.36	36.76	1,702.34
IF	3,432	132,905.56	12,500	38.73	330.47	23.77	732.53

Event	Size	Sum.	Max	Mean	S.D.	Skew.	Kurt.
Overall	6,289	501,065.24	58,000	79.67	861.38	52.91	3,348.1
BDSF	14	979.31	165	69.95	59.2	0.42	1.74
CPBP	4,046	274,245.42	10,530	67.78	284.15	17.38	509.27
DPA	1,320	144,925.39	58,000	109.79	1,679.19	31.84	1,078.95
EPWS	365	13,421.34	2,815	36.77	180.6	11.84	167.31
EDPM	53	468.69	75	8.84	17.12	2.75	9.72
EF	68	4,790.42	2,000	70.45	268.79	5.92	41.02
IF	423	62,234.67	22,518.3	147.13	1,190.73	16.6	301.77

Event	Size	Sum.	Max	Mean	S.D.	Skew.	Kurt.
Overall	1,224	238,064.49	38000	194.5	1620.89	16.74	328.28
BDSF	3	2,356.86	2,000	785.62	1,062.56	0.64	1.5
CPBP	542	121,933.77	38,000	224.97	2,106.4	15.05	245.23
DPA	467	82,631.3	20,000	176.94	1,186.33	12.76	189.43
EPWS	81	4,371.49	2,680	53.97	299.35	8.5	74.96
EDPM	39	870.38	693.05	22.32	110.41	5.97	36.78
EF	21	10,485.38	7,602.09	499.30	1,652.01	4.05	17.95
IF	71	15,415.31	8,910	217.12	1,108.67	7.16	55.34

Event	Size	Sum.	Max	Mean	S.D.	Skew.	Kurt.
Overall	1,549	118,730.18	12,447.7	76.65	426.47	18.85	482.59
BDSF	106	7,352.63	2,278.9	69.36	324.6	5.94	37.87
CPBP	637	41,966.52	5,200	65.88	289.1	11.53	174.77
DPA	549	63,056.13	12,447.7	114.86	624.34	15.33	285.82
EPWS	73	2,373.67	635.48	32.52	99.07	4.75	25.81
EDPM	95	991.84	202	10.44	27.83	4.97	30.53
EF	49	1,946.95	1,012.8	39.73	149	5.92	38.78
IF	40	1,042.44	506.97	26.06	82.33	5.21	30.68

Event	Size	Sum.	Max	Mean	S.D.	Skew.	Kurt.
Overall	1,719	116,420.51	7,900	67.73	328.12	13.68	253.1
BDSF	60	1,328.85	298	22.15	55.1	3.57	15.69
CPBP	1,254	96,366.96	7,900	76.85	355.85	13.23	237.12
DPA	64	5,107.92	1,293.38	79.81	230.44	4.11	20.23
EPWS	80	2,016.82	1,000	25.21	113.33	8.14	70.21
EDPM	76	805.39	150	10.6	23.62	4	20.87
EF	64	724.98	149.64	11.33	24.78	3.48	17.25
IF	121	10,069.59	4,260	83.22	416.52	8.69	85.52

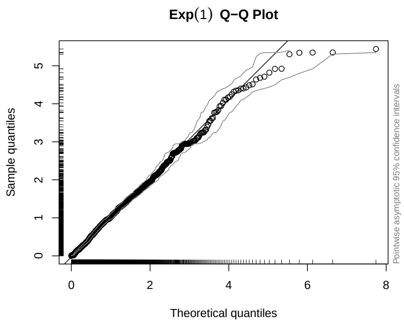
Table 2: Model selection within a Dynamic EVT-POT analysis for Financial Services and Manufacturing.

u quantile		0.92	above u #	1,149
Frequency		LRT	AIC	Selection
$log\lambda = c_\lambda$			4,027.80	
$log\lambda = f_\lambda(x)$		$\surd$	1,644.35	
$log\lambda = f_\lambda(x) + c_\lambda t$		$\surd$	1,485.74	
$log\lambda = x + h_\lambda^{(7)}(t, by\ x)$		$\surd$	710.59	$\surd$
Severity		LRT	AIC	Selection
ξ				
$\xi = c_\xi$	$\nu = c_\nu$		3,543.07	
$\xi = f_\xi(x)$	$\nu = c_\nu$	$\surd$	3,496.41	$\surd$
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu$	$\surd$	3,686.23	
ν				
$\xi = f_\xi(x)$	$\nu = c_\nu$		6,486.66	
$\xi = f_\xi(x)$	$\nu = f_\nu(x)$	$\times$	6,486.50	
$\xi = f_\xi(x)$	$\nu = c_\nu t$	$\surd$	6,483.37	$\surd$
u quantile		0.8	above u #	1,258
Frequency		LRT	AIC	Selection
$log\lambda = c_\lambda$			3,855.08	
$log\lambda = f_\lambda(x)$		$\surd$	1,313.04	
$log\lambda = f_\lambda(x) + c_\lambda t$		$\surd$	1,287.64	
$log\lambda = x + h_\lambda^{(7)}(t, by\ x)$		$\surd$	728.96	$\surd$
Severity		LRT	AIC	Selection
ξ				
$\xi = c_\xi$	$\nu = c_\nu$		6,926.07	
$\xi = f_\xi(x)$	$\nu = c_\nu$	$\surd$	4,304.00	
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu$	$\surd$	4,005.37	$\surd$
ν				
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu$		7,347.81	$\surd$
$\xi = f_\xi(x) + c_\xi t$	$\nu = f_\nu(x)$	$\surd$	7,401.31	
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu t$	$\surd$	7,404.36	

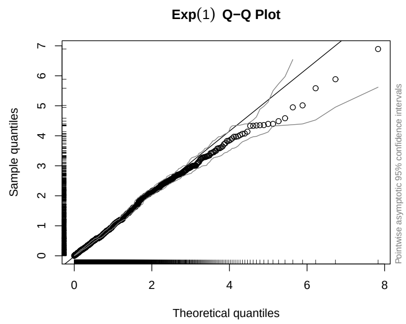
The Q–Q plots (in Figure 6) of the residuals for each industry show that the models fit the data well. The good statistical fit for different types of empirical loss data from five industries exemplifies the flexibility of the dynamic EVT-POT model. Plugging in the estimated parameters/functions one can calculate several important quantities for the underlying data. By way of example we have plotted for Financial Services and Manufacturing the frequency (see Figures 7 and 8) as well as for Financial Services the key shape parameter ξ (see Figure 9) as a function of the event type covariates. Concerning the former, frequency, we get consistent observations with those from descriptive statistics in Table 1: the most influential event type for both industries is CPBP; EF and IF are the second most influential

Table 3: Model selection within a Dynamic EVT-POT analysis for Mining, Utilities and Information.

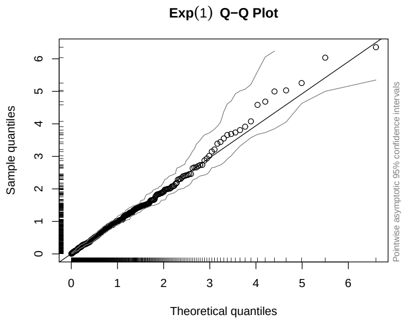
u quantile		0.7	above u #	367
Frequency		LRT	AIC	Selection
$log\lambda = c_\lambda$			1,394.94	
$log\lambda = f_\lambda(x)$		√	816.63	
$log\lambda = f_\lambda(x) + c_\lambda t$		√	665.97	
$log\lambda = x + h_\lambda^{(7)}(t, by\ x)$		√	479.86	√
Severity		LRT	AIC	Selection
ξ				
$\xi = c_\xi$	$\nu = c_\nu$		2,479.67	
$\xi = f_\xi(x)$	$\nu = c_\nu$	√	1,073.70	√
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu$	√	1,438.75	
ν				
$\xi = f_\xi(x)$	$\nu = c_\nu$		2,181.44	
$\xi = f_\xi(x)$	$\nu = f_\nu(x)$	√	2,178.54	√
$\xi = f_\xi(x)$	$\nu = c_\nu t$	×	2,178.37	
u quantile		0.92	above u #	124
Frequency		LRT	AIC	Selection
$log\lambda = c_\lambda$			513.11	
$log\lambda = f_\lambda(x)$		√	324.02	
$log\lambda = f_\lambda(x) + c_\lambda t$		√	320.91	
$log\lambda = x + h_\lambda^{(7)}(t, by\ x)$		√	77.11	√
Severity		LRT	AIC	Selection
ξ				
$\xi = c_\xi$	$\nu = c_\nu$		420.62	
$\xi = f_\xi(x)$	$\nu = c_\nu$	√	344.34	
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu$	√	338.44	√
ν				
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu$		659.30	
$\xi = f_\xi(x) + c_\xi t$	$\nu = f_\nu(x)$	×	662.97	
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu t$	√	651.79	√
u quantile		0.9	above u #	172
Frequency		LRT	AIC	Selection
$log\lambda = c_\lambda$			701.86	
$log\lambda = f_\lambda(x)$		√	404.48	
$log\lambda = f_\lambda(x) + c_\lambda t$		√	383.19	
$log\lambda = x + h_\lambda^{(7)}(t, by\ x)$		√	37.25	√
Severity		LRT	AIC	Selection
ξ				
$\xi = c_\xi$	$\nu = c_\nu$		589.93	
$\xi = f_\xi(x)$	$\nu = c_\nu$	√	585.07	
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu$	√	516.22	√
ν				
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu$		1,002.49	√
$\xi = f_\xi(x) + c_\xi t$	$\nu = f_\nu(x)$	×	1,001.50	
$\xi = f_\xi(x) + c_\xi t$	$\nu = c_\nu t$	√	1,008.41	



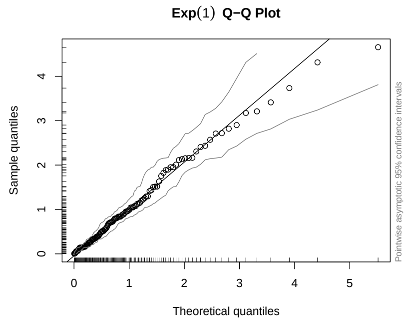
(a) Financial Services, 0.92 Quantile u



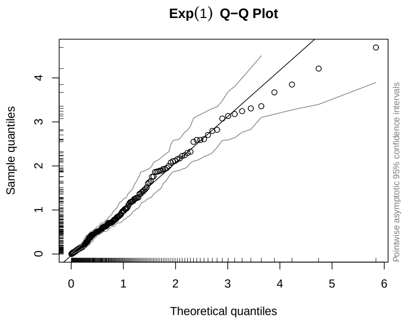
(b) Manufacturing, 0.8 Quantile u



(c) Mining, 0.7 Quantile u



(d) Utilities, 0.92 Quantile u



(e) Information, 0.9 Quantile u

Fig. 6: Q-Q plots for the residuals from the selected models (Tables 2 and 3) for Financial Services, Manufacturing, Mining, Utilities, and Information.

event types for Financial Services while DPA is the second most influential event type for Manufacturing. The frequencies of all event types for both industries are initially increasing with time and then decrease. However, the turning point for Financial Services is around 2008 while for Manufacturing it is around 2000; this is consistent with the observations made in Figure 1.

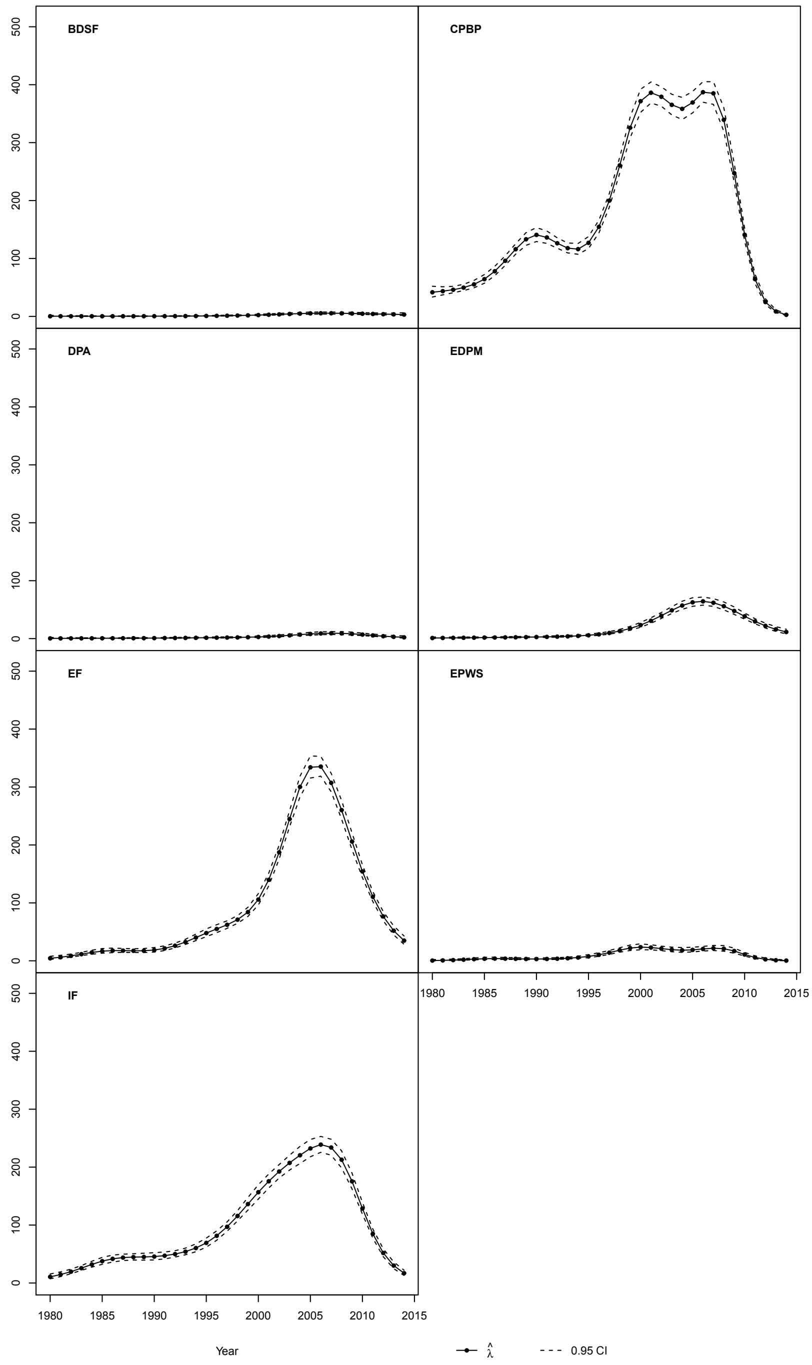


Fig. 7: $\hat{\lambda}$ with pointwise asymptotic two-sided 95% confidence intervals depending on time and event types for FS.

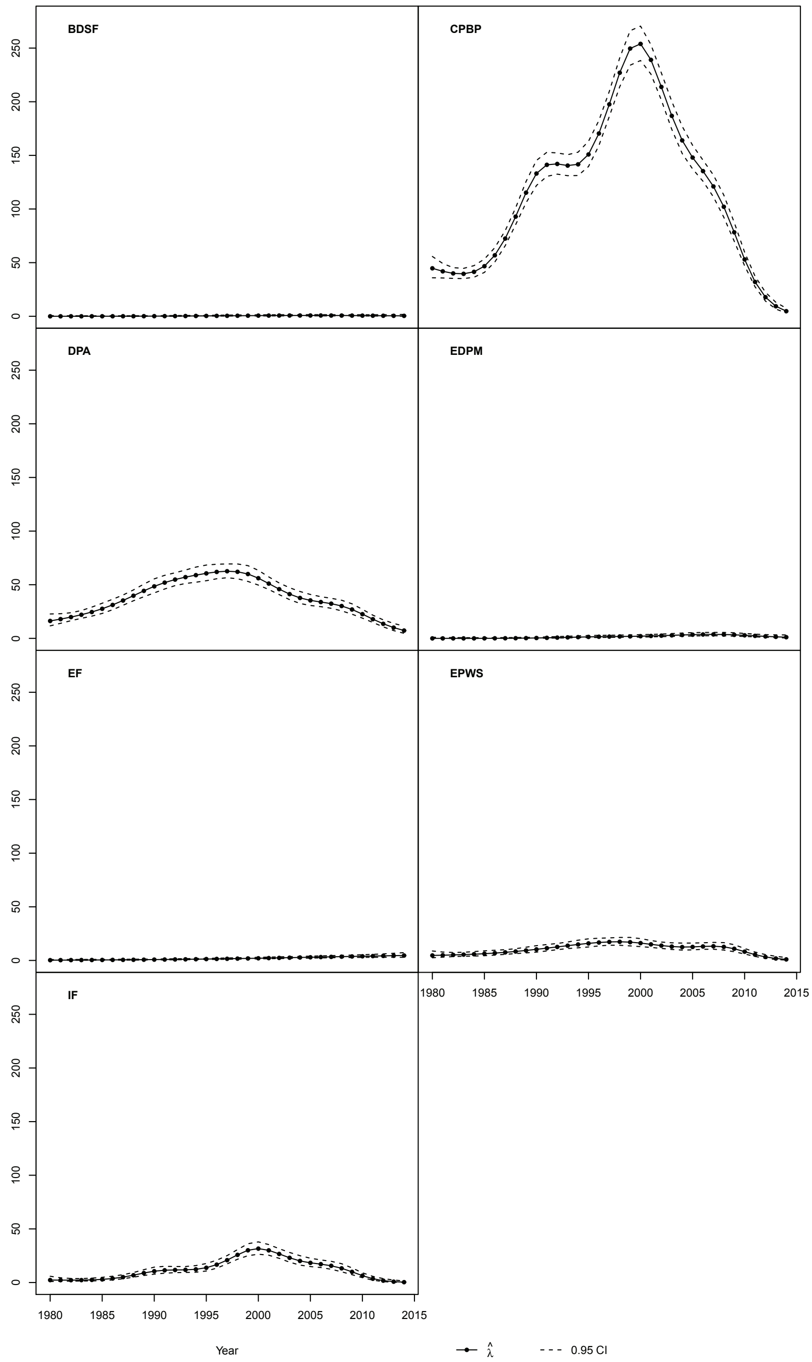


Fig. 8: $\hat{\lambda}$ with pointwise asymptotic two-sided 95% confidence intervals depending on time and event types for M.

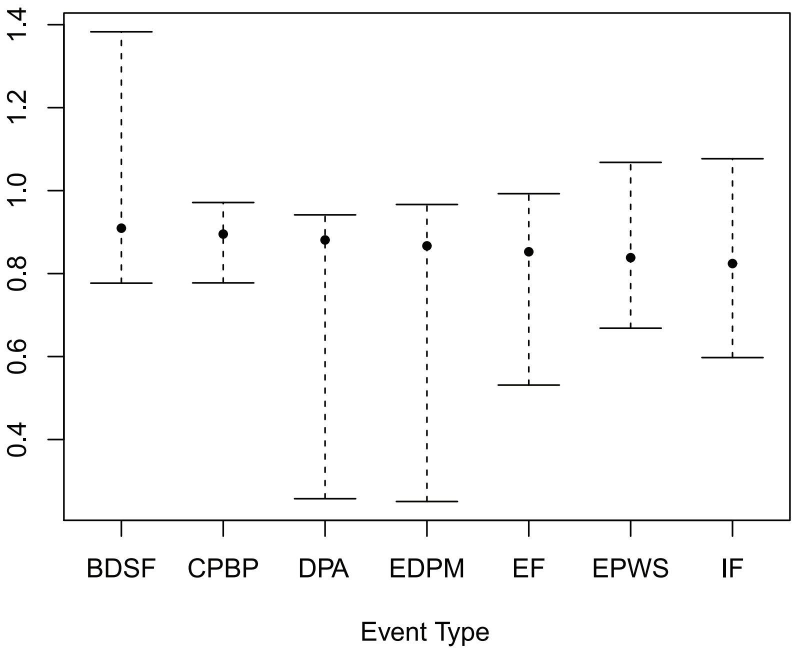


Fig. 9: $\hat{\xi}$ with bootstrapped pointwise two-sided 95% confidence intervals depending on event types for FS.

6.2 Operational Risk Severity with ICW Related Factors

Chernobai et al. (2011) have identified a strong link between operational loss frequencies and firm-specific covariates. However, due to the lack of proper models, little research has been conducted on the relation between operational loss severities and firm-specific covariates. In this section, we implement the model by Chavez-Demoulin et al. (2016) to find out whether there exists a significant influence from covariates on loss severities. We moreover estimate relevant VaR-type risk measures in function of the underlying covariates.

Doyle et al. (2007) find that firms with more ICWs tend to be smaller, younger, financially weaker, and with higher financial reporting complexity. Thus, the covariates associated with ICWs are usually selected as measures of firm size, firm age, financial health and reporting complexity. Among a number of optional factors for firm size, such as the number of transactions, trading volumes, value of assets, and net income, Chernobai et al. (2011) find the strongest correlation between operational risk frequency and market value of equity. However, this factor is only available for listed companies. In this paper (see Table 4), we use the book value of equity (Equity), asset value (Asset), the number of employees (# of Employees),

Table 4: Frequency and severity models incorporating firm specific covariates associated with ICWs.

x	λ
# of Employees	$log\hat{\lambda}(x,t) = \hat{f}_{\lambda}^{(2)}(x) + \hat{h}_{\lambda}^{(7)}(t)$ AIC: 475.768
Equity	$log\hat{\lambda}(x,t) = \hat{f}_{\lambda}^{(2)}(x) + \hat{h}_{\lambda}^{(7)}(t)$ AIC: 483.876
Net Income	$log\hat{\lambda}(x,t) = \hat{f}_{\lambda}^{(2)}(x) + \hat{h}_{\lambda}^{(7)}(t)$ AIC: 466.378
Asset	$log\hat{\lambda}(x,t) = \hat{f}_{\lambda}^{(2)}(x) + \hat{h}_{\lambda}^{(7)}(t)$ AIC: 489.794
Revenue	$log\hat{\lambda}(x,t) = \hat{f}_{\lambda}^{(2)}(x) + \hat{h}_{\lambda}^{(7)}(t)$ AIC: 481.978
Debt Ratio	$log\hat{\lambda}(x,t) = \hat{f}_{\lambda}^{(2)}(x) + \hat{h}_{\lambda}^{(7)}(t)$ AIC: 466.351
Reporting Complexity	$log\hat{\lambda}(x,t) = \hat{f}_{\lambda}^{(2)}(x) + \hat{h}_{\lambda}^{(7)}(t)$ AIC: 458.47

x	ξ	ν
# of Employees	$\hat{\xi}(x,t) = \hat{f}_{\xi}^{(2)}(x)$ AIC: 2078.61	$\hat{\nu}(x,t) = \hat{f}_{\nu}^{(2)}(x) + \hat{h}_{\nu}^{(7)}(t) + FC$ AIC: 3537.74
Equity	$\hat{\xi}(x,t) = \hat{f}_{\xi}^{(2)}(x)$ AIC: 1995.39	$\hat{\nu}(x,t) = \hat{f}_{\nu}^{(2)}(x)$ AIC: 3413.03
Net Income	$\hat{\xi}(x,t) = \hat{c}_{\xi}$ AIC: 2364.32	$\hat{\nu}(x,t) = \hat{f}_{\nu}^{(2)}(x) + \hat{h}_{\nu}^{(7)}(t) + FC$ AIC: 3531.45
Asset	$\hat{\xi}(x,t) = \hat{f}_{\xi}^{(2)}(x) + \hat{h}_{\xi}^{(7)}(t)$ AIC: 1929.24	$\hat{\nu}(x,t) = \hat{f}_{\nu}^{(2)}(x)$ AIC: 3500.84
Revenue	$\hat{\xi}(x,t) = \hat{f}_{\xi}^{(2)}(x)$ AIC: 1834.91	$\hat{\nu}(x,t) = \hat{f}_{\nu}^{(2)}(x) + \hat{h}_{\nu}^{(7)}(t) + FC$ AIC: 3394.62
Debt Ratio	$\hat{\xi}(x,t) = \hat{c}_{\xi}$ AIC: 1492.88	$\hat{\nu}(x,t) = \hat{f}_{\nu}^{(2)}(x)$ AIC: 2791.54
Reporting Complexity	$\hat{\xi}(x,t) = \hat{f}_{\xi}^{(2)}(x) + \hat{h}_{\xi}^{(7)}(t) + FC$ AIC: 1390.53	$\hat{\nu}(x,t) = \hat{f}_{\nu}^{(2)}(x)$ AIC: 2924.54

x	quantile	u	n
# of Employees	90%	75.8	662
Equity	90%	78	640
Net Income	90%	78.841	634
Asset	90%	78.48	638
Revenue	90%	77	633
Debt Ratio	92%	102.6136	510
Reporting Complexity	92%	102.18	548

Net Income, and Revenue to represent firm size. Larger firms tend to have more mature internal controls but also higher trading volumes as well as more complex transactions and organizational structures; therefore intuitively firm size tends to have positive impacts on operational risk, which is contrary to the conclusion in Doyle et al. (2007). As was exemplified throughout the financial crisis, and indeed also follows more methodologically from the Merton structural model for default,

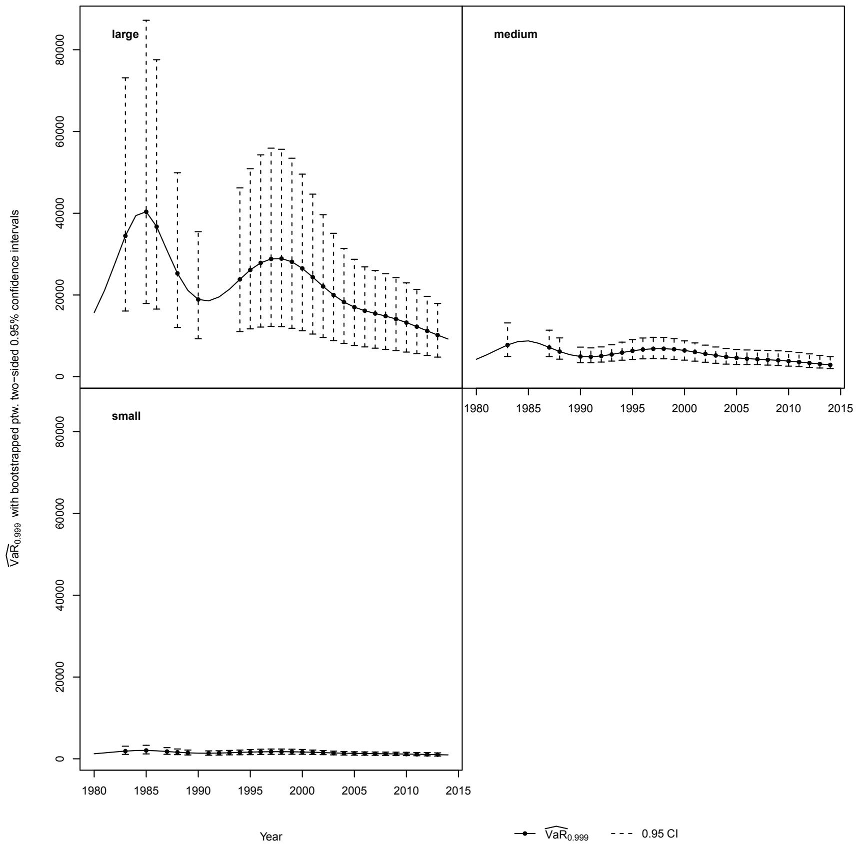


Fig. 10: $\widehat{VaR}_{0.999}$ with bootstrapped 95% confidence intervals depending on levels of firm size (represented by Revenue) for all the industries from 1980 to 2015.

leverage plays a key role as relevant factor underlying financial distress. As a proxy for leverage, we include Debt Ratio as one of the covariates. Finally, it is clearly more difficult to monitor and control firms with higher financial reporting complexity. Following the factors used in Doyle et al. (2007), we identify firms with multinational segments and foreign currency translations as the ones with high Reporting Complexity.

Using the factors above to measure firm size, financial health, and reporting complexity, we fit dynamic EVT models incorporating these covariates. To assess the impact of the financial crisis, we also include a dummy variable (FC) for the crisis. The models selected are shown in Table 4. The Q-Q plots of the residuals for the severity models including the ICW covariates show a good overall fit (plots not shown). All the frequency models depend on covariates and time, which is consistent with the conclusion of Chernobai et al. (2011). However, none of the frequency models include FC. Except for the models on Net Income and Debt

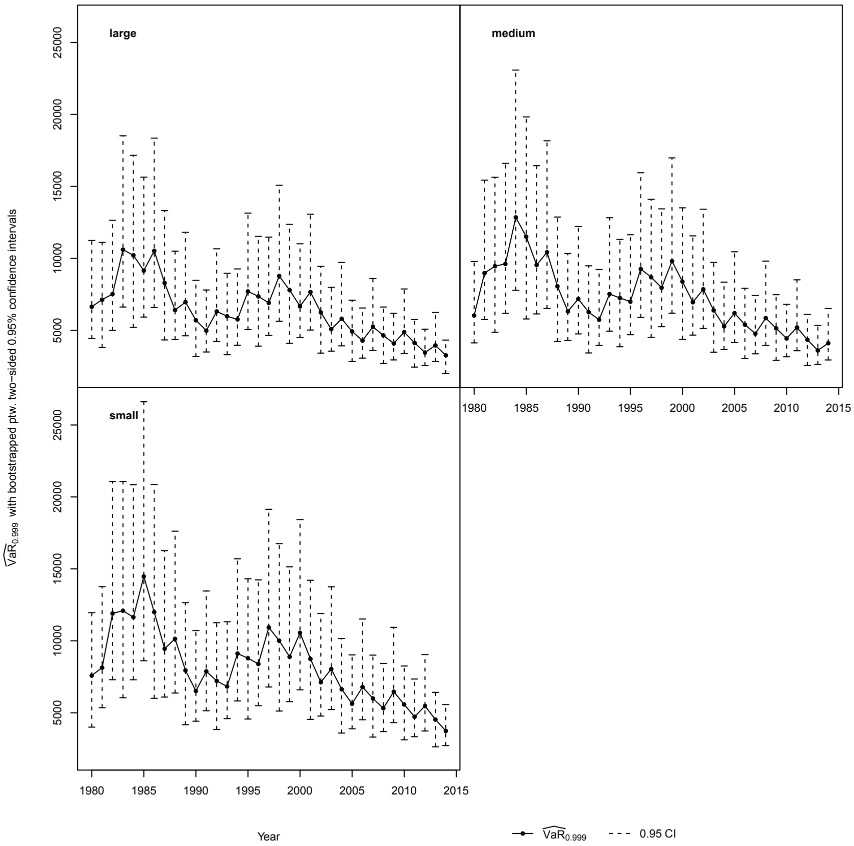


Fig. 11: $\widehat{VaR}_{0.999}$ with bootstrapped 95% confidence intervals depending on levels of financial health (represented by Debt Ratio) for all the industries from 1980 to 2015.

Ratio, all the other models for ξ depend on covariates associated with ICWs; all models for ν depend on covariates associated with ICWs. The dummy variable FC is significant in severity models on the Number of Employees, Net Income, Revenue and Reporting Complexity, which highlights the obvious fact that there is a strong impact of the financial crisis on operational risk. Among all the covariates controlling for firm size, i.e., Equity, Asset and Revenue, Revenue achieves the lowest AIC among all severity models.

We use the data including all the industries from year 1980 to 2015 for the analyses in this section. According to the 1/3 - and 2/3 - quantiles of the values for the chosen covariates, we divide the loss frequencies and severities into three groups (small, medium, and large) corresponding to each covariate. We use Revenue as the controlling covariate for firm size, Debt Ratio as the controlling covariate for Financial Health, and Reporting Complexity defined by whether or

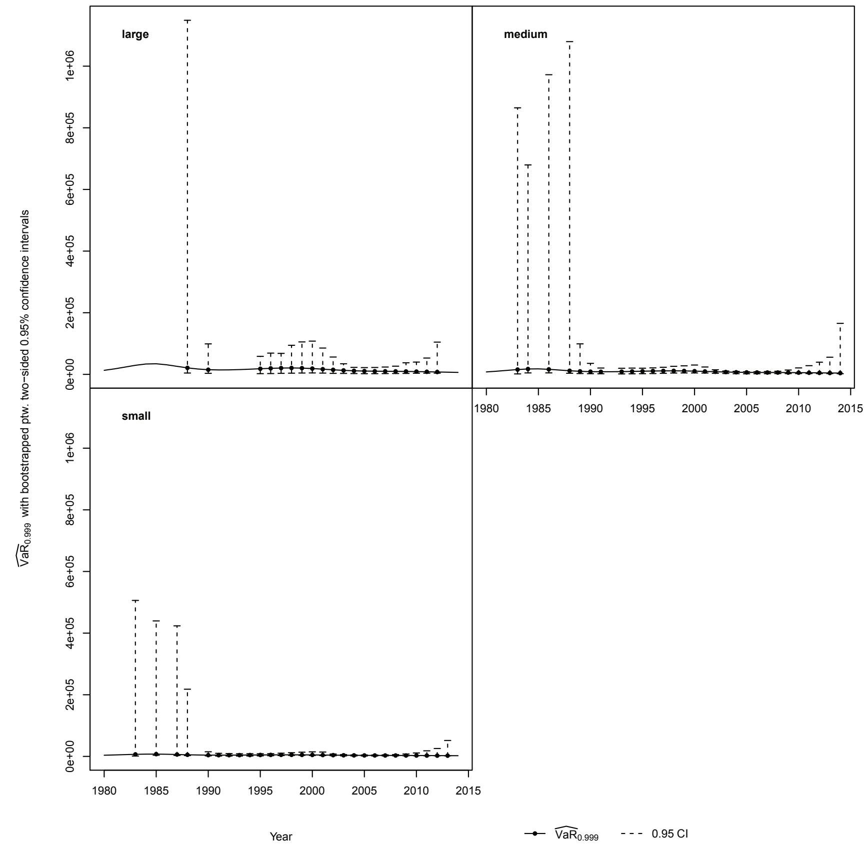


Fig. 12: $\widehat{VaR}_{0.999}$ with bootstrapped 95% confidence intervals depending on levels of Reporting Complexity for all the industries from 1980 to 2015.

not there is inclusion of global segments and foreign currency translations. The dynamic POT model is fitted for each of the above covariates, and the estimated risk measurements (99.9% VaR) for each group are shown in Figures 10, 11 and 12. From the figures we can observe that the firms with larger size (represented by revenue) and higher reporting complexity result in larger estimated risk measures for operational risk severity ($\widehat{VaR}_{0.999}$). The result on the relation between operational risk and Debt Ratio tends to be opposite to the conclusion of Doyle et al. (2007). The potential reason is that the debt ratios for the data sample are mainly less than 1, which is clearly less than the level that will cause debt overhang issues. Instead, reasonable leverage could be helpful for a firm's business operations.

The models fitted in Table 4 clearly show that both frequency and severity, as well as the estimated measures of risk (VaR), of operational risk losses show a strong link with the covariates associated with ICWs.

7 Conclusion

In the seminal paper Dutta and Perry (2007), the authors present key criteria which any quantitative approach for the modeling of operational risk should satisfy: Good Fit, Realistic, Well-Specified, Flexible and Simple. Now, ten years later, these criteria still very much hold. Things that have changed since then are the loss of momentum in the AMA-LDA approach towards the calculation of regulatory capital, and the gradual improvement of available (though rarely public) data sources. From the regulators' point of view, we clearly observe a shift away from Internal Models towards Standardized ones. On the other hand, the 2007-2009 financial crisis has amply shown the increasing relevance of operational risk as an important risk class, and this not only for the financial industry. As a consequence, at least for internal quality control purposes, industry is well advised to statistically measure as well as corporate governance wise understand the threat posed to its business by operational risk. On the basis of an industry wide data set, in this paper we analyze operational risk losses (both frequency as well as severity) using a so-called dynamic EVT-POT approach. Besides presenting some more detailed statistical analyses of operational risk losses for specific industries, we also compare and contrast losses between financial and non-financial firms.

Looking in Dutta and Perry (2007) at the definitions of the above criteria, it is fair to say that the methodology presented in our paper satisfies the requirements set for Good Fit, Well-Specified and Flexible. For Realistic, these authors write: "If a model fits well in a statistical sense, does it generate a loss distribution with a realistic capital estimate?" This is always very difficult, if not impossible to answer from our position as outside researchers. As we discussed at various points in the paper, the statistical frequency-severity fit across industries, taking ICW covariates into account, is meaningful. Our results suggest that firms with larger size and higher reporting complexity are characterised by larger operational risk VaR values. We do not find support for the same relation between the firms' financial health and operational risk. However, the estimated 99.9% VaR measures do follow the empirical loss values as well as the underlying economic cycles well (see Figures 5, 10-12). To conclude from there that risk capital estimates are realistic is not really possible. In any case, within the realm of banking, such a conclusion becomes less relevant under a regulatory regime framework moving away from the AMA-LDA. More importantly, our data fitting algorithms aim at providing a better understanding of the underlying data especially in function of specific covariates.

This brings us to the final criterion of Simplicity: it is fair to say that the Dynamic EVT-POT approach developed by Chavez-Demoulin et al. (2016) and applied in this paper is not a particularly simple one. If however one is interested in modeling power-type data far in the tail (and that surely is one of the important achievements of EVT), and at the same time non-stationarity and covariate inclusion are relevant, then our approach is close to canonical. Moreover, the method discussed is fully supported by an R-library providing all the relevant algorithms; see Chavez-Demoulin et al. (2016). We therefore strongly believe that these techniques offer a useful tool for industry internal and regulatory modeling of operational risk data and at the same time it addresses the calls from the industry for a new class of exposure based models that capture the dynamic nature of operational risk losses (see Farha et al. 2016). For instance, based on the fitted models it is easy to simulate stress scenarios taking certain regime switches (i.e., special covariates) into account. A future combination of machine learning technology with EVT based fitting could no doubt prove useful; for this to happen, one would of course need larger (and better) data sets. Furthermore, other areas of risk modeling can also benefit from the proposed methodology. Catastrophe risk models and credit risk measurement are only two examples that warrant further investigations into the application of the dynamic EVT-POT approach.

Acknowledgements The authors would like to thank Valerie Chavez-Demoulin for helpful comments during the early stages of the project. We also acknowledge the comments from Gerhard-Wilhelm Weber. Part of this paper was written while PE was Hung Hing Ying Distinguished Visiting Professor of Science and Technology in the Department of Statistics and Actuarial Science at the University of Hong Kong.

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