

# Cover times in the discrete cylinder

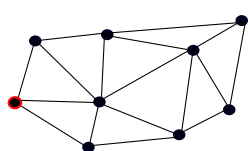
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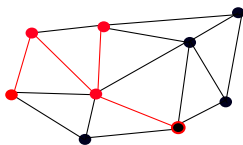
La Pietra Meeting, June 2011

# What is a cover time?

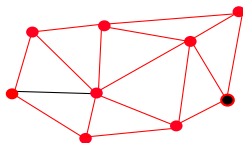
Illustration of cover time  $C$ :



$t=0$



$t=5$



$t=15, C=15!$

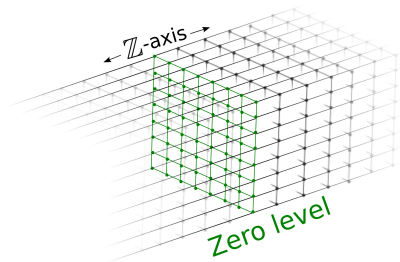
- ▶  $G_1, G_2, G_3, \dots, |G_N| \rightarrow \infty$  as  $N \rightarrow \infty$ .  $C_N$  cover time of  $G_N$ .
- ▶ Often similar behavior to coupon collector with  $|G_N|$  coupons, that is  $\mathbb{E}C_N \sim c|G_N| \log |G_N|$  and even  $\frac{C_N}{c|G_N| \log |G_N|} \xrightarrow{\text{Prob. 1}}$  1.
- ▶ Difficult question: Do fluctuations have Gumbel distribution (CDF  $F(z) = e^{-e^{-z}}$ ), i.e. does

$$\frac{C_N}{c|G_N|} - \log |G_N| \xrightarrow{\text{Law}} \text{Gumbel?}$$

Unknown for e.g.  $G_N = d$ -dimensional discrete torus,  $d \geq 3$ .

## Gumbel limit for cover times in cylinder

The Discrete Cylinder graph,  $G_N = (\mathbb{Z}/N\mathbb{Z})^d \times \mathbb{Z}, N \geq 1$ :



Let  $L_t$  be local time at zero of  $\mathbb{Z}$ -component of random walk.

### Theorem

([2], B. 2011, Special case)  $C_N$  cover time of zero level

$\mathbb{T}_N = (\mathbb{Z}/N\mathbb{Z})^d \times \{0\}, d \geq 2$ . Have

$$\frac{L_{C_N}}{cN^d} - \log N^d \xrightarrow{\text{Law}} \text{Gumbel distribution as } N \rightarrow \infty.$$

# References



Aldous & Fill

*Reversible Markov Chains and Random Walks on Graphs*,  
Chapters 6-7.

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*Cover times in the discrete cylinder.*

arXiv:1103.2079. Submitted.



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*Vacant set of random interacements and percolation*

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