PRO*DOC/IRTG STOCHASTIC MODELS OF COMPLEX PROCESSES
SUMMER SCHOOL 2010 IN DISENTIS

Exit distributions of Random Walks in Random Environments in $\mathbb{Z}^d$, $d \geq 3$.

Erich Baur, UZH

We present a result of Bolthausen and Zeitouni (2007). They show that in $\mathbb{Z}^d, d \geq 3$, the exit measures from large balls of random walks in i.i.d. random environments, which are small perturbations of the environment corresponding to simple random walk, are in a suitable sense close to the exit measure of simple random walk.

E. Bolthausen and O. Zeitouni. *Multiscale analysis for exit distributions of random walks in random environments.* Probability Theory and Related Fields. 138, 581-645, 2007.

Asymptotic Support Theorem for Planar Isotropic Brownian Flows.

Moritz Biskamp, TU Berlin

First we will introduce the notion of stochastic flows and isotropic Brownian flows (IBF) in particular, which are basically random diffeomorphisms in $\mathbf{R}^d$, whose image of any single point is a Brownian motion, and for which the covariance tensor between two Brownian motions is an isotropic function of their position. It has been shown by various authors that the diameter of some given non-trivial bounded set $\mathcal{X}$ grows linearly in time under the action of the flow, provided it has a non-negative top-Lyapunov exponent. In case of planar IBFs with a positive top-Lyapunov exponent the precise deterministic linear growth rate K is known. In this talk we will present the ideas to extend this result an asymptotic support theorem for the linearly time-scaled trajectories of a planar IBF – meant in the sense that the set of time-scaled trajectories converge in probability in the Hausdorff distance to the set of Lipschitz functions starting in 0 with Lipschitz constant K.

Cover Levels and Random Interlacements.

David Belius, ETHZ

In the random interlacements model, the cover level of a finite subset A is the least level such that the set is completely contained in the random interlacement at that level. I will present a result that shows that if you rescale and recenter the cover level appropriately its distribution tends to the Gumbel distribution as the cardinality of A tends to infinity. Since the random interlacements model in a certain sense models the trace of a random walk in the discrete torus or in the discrete cylinder, this result is related to cover times of sets by simple random walk in these graphs. For cover times the corresponding distributional limit result is only a conjecture.

Time-consistent mean-variance portfolio selection in discrete and continuous time.

Christoph Czichowsky, ETHZ

In a general semimartingale setting we study mean-variance portfolio selection under the aspect of time consistency. Since the usual formulation used in dynamic programming is time inconsistent in the sense that the dynamic programming principle fails, we have to develop a different dynamic formulation in order to obtain a time-consistent solution. We start in discrete time, where the formulation is straightforward, and then find the natural extension to continuous time. This generalises recent results by Basak and Chabakauri (2007) and Björk and Murgoci (2008) where the treatment relies on an underlying Markovian framework. As a new feature we justify the continuous-time formulation by showing that it coincides with the continuous-time limit of the discrete-time formulation. As mean-variance portfolio selection is one of the classical problems in mathematical finance we introduce and discover several classical notions used in the field such as the structure con dition, the mean-variance tradeoff process, the Föllmer-Schweizer decomposition and local risk minimisation.along the way. To make the talk accessible for a broad audience we try to give a self-content presentation that does not assume previous knowledge about mathematical finance.

BSDEs with rough generators.

Joscha Diehl, TU Berlin

Classically, the driver of a backward stochastic differential equation (BSDE) is a Lipschitz continuous function that is integrated with respect to Lebesgue measure. We introduce a new class of BSDEs where the driver consists of an additional integral in the rough path sense. We show existence, uniqueness and stability results for these equations. There is a close connection to stochastic partial differential equations driven by rough paths. This is work in progress with Peter Friz (TU Berlin).

Poisson approximation for extreme values of univariate and bivariate geometric random variables.

Anne Feidt, UZH

We approximate the distribution of the maximum of an i.i.d. sample of geometric random variables by a Poisson distribution and study the accuracy in total variation of this approximation using the Stein-Chen method. This improves an asymptotic result for the distribution of the maximum by Nadarajah and Mitov (2002). An attempt to generalize to higher dimensions has led us to consider an example of a bivariate geometric distribution with dependent components: the bivariate Marshall-Olkin geometric distribution. We search for a Poisson process that will give a good approximation of the point process that counts simultaneous extreme values of both components of Marshall-Olkin geometric pairs.

Long Memory in a Linear Stochastic Volterra-Delay Differential Equation.

Katja Krol, HU Berlin

In this talk we consider the following stochastic Volterra-delay equation:

$$dX(t) = \left(\int_0^t k(t-s)X(s)\,ds - \int_{-\tau}^0 X(t+u)a(\,du) \right) dt + \sigma\,dW(t), \quad t > 0,$$

where k is an integrable continuous kernel, a a finite positive Borel measure on $[-\tau, 0]$ and W represents a standard Brownian motion. Since a and k represent short-term and long-term weights respectively we require $\int_0^\infty k(t)\,dt = a[-\tau, 0]$.

For completely monotone, regularly varying kernels k satisfying some additional conditions, we determine the exact asymptotic behavior of the fundamental solution and show that if it is stable then it decays polynomially to zero. In this case there exists an asymptotically stationary solution. The autocovariance function of the stationary process is also regularly varying at infinity and its exact pointwise rate of decay can be determined. Moreover, it can be shown for certain kernels that this stationary process has long memory in the sense that the autocovariance function is not integrable over the reals. This is based on a joint work with John A. D. Appleby, Dublin City University.

Affine processes are regular.

Martin Keller-Ressel, ETHZ

Affine Processes are a class of Markov processes with important applications in finance and other fields. These processes have been described and fully characterized by Duffie, Filipovic and Schachermayer (2003) under the condition of 'regularity'. Here, a Markov process is called regular, if its characteristic function is differentiable in time with (space-)continuous derivatives. In joint work with Walter Schachermayer and Josef Teichmann I have shown that for affine processes this condition is automatically fulfilled, i.e. that every affine process is regular. The regularity problem has interesting connections to Hilbert's fifth problem on the differentiability of continuous transformation groups, from which we borrow some tools that are unusual in the field of stochastics. Recently the technique of our proof has been successfully applied to show regularity of matrix-valued affine processes.

On a flow of operators associated to virtual permutations.

Joseph Najnudel, UZH

In this talk, we study random infinite sequences of permutations of increasing finite order, which satisfy some natural properties of compatibility for their cycle structure. We deduce an explicit description of the global cycle structure of these sequences, and from this description, we construct some random flows of operators, acting on a suitable functional space

Time correlation for the parabolic Anderson model.

Adrian Schnitzler, TU Berlin

We consider asymptotics for the time correlation of the parabolic Anderson model, i.e. the Cauchy problem for the heat equation on the lattice with random potential. We show how to derive exact formulae in the case of a potential that consists of an i.i.d. field of nonnegative random variables with tails that decrease more slowly than those of a double-exponential distribution. Furthermore, we apply these results to investigate intermittency and ageing properties of the model.

Random quantization of probability measures on $\mathbb{R}^d$.

Reik Schottstedt, TU Berlin

We treat the approximation of a probability measure μ on $\mathbb{R}^d$, $d \geq 3$, by its empirical measure $\hat{\mu}_N$ (interpreted as random quantizer) in the Wasserstein-metric. This problem arises for instance in weak approximation of SDEs. We show that the expected p-th power ($p \geq 1$) of the Wasserstein-metric of μ and $\hat{\mu}_N$ generated by N i.i.d. samples of μ converges to zero with constant times optimal rate $N^{-\frac{p}{d}}$ as N goes to infinity. Further we show the existence of the constant.

Convergence of Monte Carlo approximations (according to the Bender/Denk algorithm, 2007) for Lipschitz BSDEs in Processes Space: a first proof and formulation of the error.

Plamen Turkedjiev, HU Berlin

The Bender/Denk algorithm is standard tool for the approximation of Lipschitz BSDEs driven by Brownian Motion. This builds a time discretized approximation in three stages, each associated to a different mathematical object: first using conditional expectations; second using projections on finite dimensional spaces to estimate the conditional expectations; finally using Monte Carlo estimates of the projections.

The convergence of the algorithm is well known in each of the first two stages, but in the Monte Carlo stage only almost sure convergence at the discretization points is known. To our knowledge, there is no known estimate of the standard error in process space for the full scheme. This is an important measure of the rate of convergence of the numerical scheme to the true solution. In our work, we establish an upper bound for the standard error, and use this to establish a rate of convergence for the algorithm to the true solution. Thereby, we complete the work of the original paper in the sense that we extend the understanding of the numerical convergence to that of the competing algorithm introduced by Gobet, Lemor and Warin 2006.

The aim of this presentation is to outline the derivation of the upper bound for the standard error and to present the basic tools that we use. We also discuss some current open problems associated with basis selection, and demonstrate some numerical simulations.