Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

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| **Family name:** | **Department:** |
| **First name:** | **ETH ID No.:** |

For the grading:

| | **1K** | **2K** | **Points** | **Comments:** |
|---|---|---|---|---|
| **1** | | | | |
| **2** | | | | |
| **3** | | | | |
| **4-13** | | | | |
| **Total** | | | | |

# MATHEMATICS I EXAM

**for students of Agricultural Science, Earth Sciences, Environmental Sciences, and Food Science**

**Important:**

- Please fill the header on the cover page and lay your ETH-card visible on the table.
- Please write neatly with a non erasable blue or black pen, in particular not with a pencil. Beware that something that is too hard to read could be ignored.
- Please leave some empty space on the margins for the correction.
- This exam has 13 questions and lasts for 90 minutes.

**For questions 1-3:**

- Please write down all intermediate steps of your calculations and solutions.
- Write your name and ETH ID / Legi-Nr. on each additional sheet.
- The maximal score of each exercise part is given in the right margin.

**For questions 4-13:**

- Mark your answers clearly.
- There is always only one correct answer and 2 points per question.

**Permitted aids:**

- Written notes up to 20 A4-Pages, one English dictionary,
- **no** calculator, **no** mobile phone, **no** laptop.
- Please switch off your mobile phone and stow it away.

Good Luck!

**1.** Consider the function

$$f(x) = \sqrt{x^2+5} \quad \text{for } x \in \mathbb{R}.$$

**a)** Determine the linearization of $f(x)$ in $x_0 = 2$. 4 points

**b)** Determine the range of $f(x)$. 3 points

**c)** Let $F(x)$ be the solution of the initial value problem

$$\begin{cases} F'(x) = f(x) \\ F(0) = 33. \end{cases}$$

Is $F(1)$ bigger or smaller than $33$? You do not have to compute $F(x)$. 3 points

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**2.** Determine the general solution of each of the following differential equations:

**a)** $y'' = 6y' - 10y$ 5 points

**b)** $3y' = (y-1)(y+2)$ 5 points

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**3.** Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 4 & 4 & 4 \\ 0 & 0 & 1 & 2 & 3 \end{pmatrix}.$$

**a)** Is the system

$$A\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

solvable? 3 points

**b)** Determine a basis of the solution set of the matrix equation $A\vec{x} = \vec{0}$. 4 points

**c)** Determine a basis of the space of all vectors $\vec{v}$ for which the matrix equation $A\vec{x} = \vec{v}$ is solvable. 3 points

**For exercises 4-13:** Each question gives $2$ points. Wrong or multiple answers give $0$ points. Mark your answers on this exam.

**4.** The determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 0 & 1 \\ 4 & 5 & 1 & 3 \\ 2 & 6 & 0 & 0 \end{pmatrix}$$

is

(a) $-2$.

(b) $-1$.

(c) $1$.

(d) $2$.

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**5.** The vector $\vec{v} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ is an eigenvector of the matrix

$$\begin{pmatrix} 4 & 1 & 1 \\ -5 & 0 & -3 \\ -1 & -1 & 2 \end{pmatrix}.$$

What is the eigenvalue belonging to $\vec{v}$?

(a) $-2$

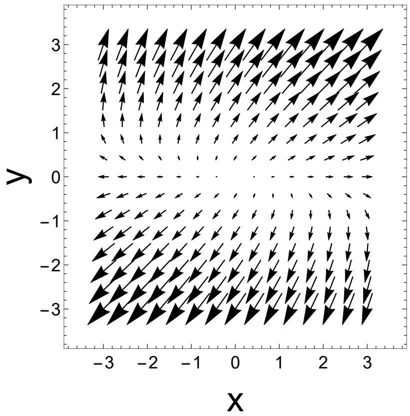
(b) $-1$

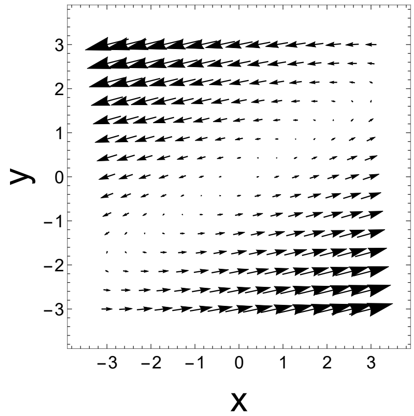
(c) $1$

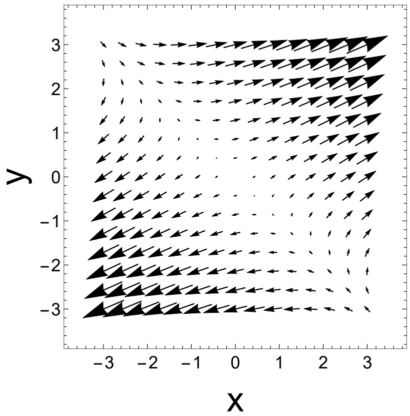
(d) $2$

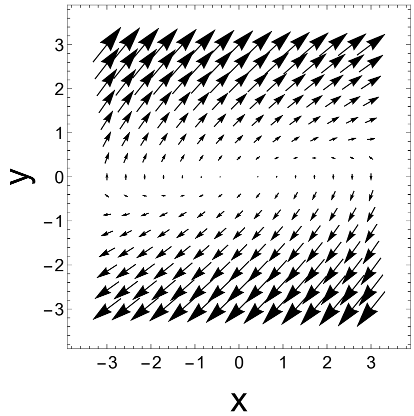
**6.** Which picture shows the phase portrait of the system

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix} \vec{x} \quad ?$$

(a) 


(c) 


(b) 


(d) 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**7.** Consider the discrete system

$$\begin{cases} a(N+1) = 3a(N) + 4b(N) \\ b(N+1) = 2a(N) + b(N) \end{cases} \qquad \text{mit } N = 0, 1, 2, \dots$$

For which initial value does the solution Lösung $\vec{x}(N) = \begin{pmatrix} a(N) \\ b(N) \end{pmatrix}$ of the corresponding initial value problem stay bounded?

(a) $\begin{cases} a(0) = -1 \\ b(0) = 1 \end{cases}$ (b) $\begin{cases} a(0) = 2 \\ b(0) = 0 \end{cases}$ (c) $\begin{cases} a(0) = 2 \\ b(0) = 1 \end{cases}$ (d) $\begin{cases} a(0) = 0 \\ b(0) = -1 \end{cases}$

**8.** The limit

$$\lim_{x \to +\infty} x^{\frac{2}{x}}$$

is given by

(a) $0$

(b) $1$

(c) $2$

(d) $+\infty$

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**9.** Consider the function

$$f(x) = \int_0^x \ln(t^2 + e^3)\, dt.$$

Then $f'(0)$ is given by

(a) $0$

(b) $1$

(c) $2$

(d) $3$

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**10.** Let $g(y)$ be the inverse function of the function

$$y = f(x) = e^{(3-x)^3 - 1}.$$

Consider $g(y)$ at the point $y = f(2) = 1$. Then the derivative $g'(1)$ is given by

(a) $-3$

(b) $-\frac{1}{3}$

(c) $\frac{1}{3}$

(d) $3$

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**11.** The expression $\frac{3+i}{2-i}$ is equal to:

(a) $\sqrt{2}e^{-i\frac{\pi}{3}}$

(b) $\sqrt{2}e^{-i\frac{\pi}{4}}$

(c) $\sqrt{2}e^{i\frac{\pi}{4}}$

(d) $\sqrt{2}e^{i\frac{\pi}{2}}$

**12.** The zeros of the polynomial $p(\lambda) = \lambda^4 + 1$ are given by:

(a) $-1, 1, -i, i$

(b) $-1, 1, e^{i\frac{\pi}{4}}, e^{-i\frac{\pi}{4}}$

(c) $e^{i\frac{\pi}{4}}, e^{i\frac{3\pi}{4}}, e^{i\frac{5\pi}{4}}, e^{i\frac{7\pi}{4}}$

(d) $e^{i\frac{\pi}{4}}, e^{i\frac{5\pi}{4}}, -i, i$

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**13.** Let $f(x)$ be a function that is differentiable for all $x \in \mathbb{R}$. Which of the following statements are always true?

(I) If $f$ is not injective, then there exists a $c$ with $f'(c) = 0$.

(II) If there exists a $c$ with $f'(c) = 0$, then $f$ is not injective.

(a) Both statements (I) and (II) are true.

(b) Statement (I) is true, but statement (II) is false.

(c) Statement (II) i true, but statement (I) is false.

(d) Both statements (I) and (II) are false.