# Tartar's conjecture and localization of quasiconvex hulls

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We consider the issue of compactness for sequences of approximate solutions to differential inclusions of the type

$$Du(x) \in K \text{ a.e.}$$

for functions $u : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2$, where $K$ is a prescribed compact set of $2 \times 2$ matrices. It is well known that a necessary condition for compactness is that $\operatorname{rank}(A - B) \geq 2$ for any two distinct $A, B \in K$ (i.e. the set $K$ should contain no *rank-one connections*), and in 1982 L.Tartar conjectured that this condition might be also sufficient. In fact subsequently Tartar himself produced an example of a set $K$ consisting of 4 matrices where there are no rank-one connections but compactness fails. Such examples are nowadays called $T_4$ *configurations*. On the other hand the original conjecture was verified by V.Šverák in 1993 for *connected* sets $K \subset \mathbb{R}^{2\times 2}$.

Our main result is that if $K$ is a general compact set which contains no rank-one connections and no $T_4$ configurations, then sequences of approximate solutions are compact (i.e. the set $K$ supports no non-Dirac gradient Young measures). We also show that, in contrast to the situation in higher dimensions, the quasiconvex hull of compact sets in $2 \times 2$ can be *localized* in the sense that if $K^{qc} \subset \bigcup_{i=1}^{N} U_i$ for pairwise disjoint open sets $U_i$, then $K^{qc} \cap U_i = (K \cap U_i)^{qc}$.

Our proof goes via a careful analysis of the rank-one geometry of $\mathbb{R}^{2\times 2}$, combined with the use of Beltrami operators and the theory of quasiregular mappings in the plane to obtain separation results for homogeneous gradient Young measures.

This is joint work with Daniel Faraco from Madrid.