Galerkin-type approximation of the HJM forward interest rates dynamics and applications to the analytical pricing of American Bond Options

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We study the problem of pricing an American Put Option on a Zero Coupon Bond when the market is described by the celebrated Heath-Jarrow-Morton model [1] and the underlying process is the forward interest rate in the Musiela parametrization [2]. Then pricing the American Bond Option gives rise to an optimal stopping problem of a Hilbert space-valued process. Such a problem was investigated by Gatarek and Święch [3] in the framework of viscosity solutions and by Zabczyk [4] and Barbu, Marinelli [5] in terms of excessive measures.

We construct an analytical approximation scheme for the infinite dimensional diffusion that allows to obtain the price of the Bond Option in terms of a sequence of finite dimensional optimal stopping problems. Each one of them corresponds to the price of an American Barrier Option and may be treated by standard methods of variational inequalities in $\mathbb{R}^n$. The sequence of approximating value functions converges to the original one uniformly on bounded subsets of the Hilbert space. A characterization of the continuation and stopping regions of the original problem is then obtained in terms of the limiting behavior of the approximating continuation and stopping regions.

The results obtained in this framework seem to be easily extendable to a large number of cases of interest in SPDE theory.

References

[1] D. Heath, R. Jarrow, A. Morton, *Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation*, Econometrica **60** (1) (1992): 77-105.

[2] M. Musiela, *Stochastic PDEs and term structure models*, Journées Internationales de Finance, IGR-AFFI, La Baule (1993).

[3] D. Gątarek, A. Święch, *Optimal Stopping in Hilbert Spaces and Pricing of American Options*, Math. Methods of Operations Research **50** (1999): 135-147 .

[4] J. Zabczyk, *Bellman's Inclusions and Excessive Measures*, Probability and Mathematical Statistics **21** (1) (2001): 101-122.

[5] V. Barbu, C. Marinelli *Variational Inequalities in Hilbert Spaces with Measures and Optimal Stopping Problems*, Applied Mathematics and Optimization **57** (2008): 237-262.