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Dynamic Risk Measures and Bid-Ask Dynamic Pricing

The key notion for dynamic risk measures is the time consistency. We characterize the time consistency by a "cocycle condition" of the minimal penalty function. Under a mild condition we prove that the dynamic risk process associated with any financial instrument has a cadlag modification. Applying the cocycle condition, we construct new families of time consistent dynamic risk measures using the theory of right continuous BMO martingales. These examples generalize the solutions of Backward Stochastic Differential Equations and lead to processes with cadlag paths which may have jumps.

We apply the time consistent dynamic risk measures to pricing in financial market taking into account both transaction costs and liquidity risk. We construct a procedure which assigns to each financial instrument a Dynamic Bid Price Process and a Dynamic Ask Price Process in such a way that it is compatible with the price dynamics of some reference securities and also with the observed bid and ask prices at time zero for some reference options. Such procedures lead to an arbitrage free dynamic price system. We extend to that context the fundamental theorem of asset pricing.