Ergodic Control, Ergodic PDE and Ergodic BSDE

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Abstract

This is a joint work with Marco Fuhrman and Gianmario Tessitore.

It is well known that there exists a link among control, PDE and BSDE.

Ergodic control problem is:

$$\inf_v J(x,v) = \inf_v \limsup_{T\to\infty} \frac{1}{T} E \int_0^T l(X_s^x, v_s) ds,$$

where

$$dX_t^x = F(X_t^x)dt + G(dW_t + r(v_t)dt), \; X_0^x = x,$$

and v is a control process taking values in U.

It is known that the PDE (called ergodic PDE) associated with this ergodic control problem is:

$$Lu + \psi(x, G\nabla u) - \lambda = 0,$$

where

$$\psi(x,z) = \inf_{v\in U}\{l(x,v) + zr(v)\},$$

and L is the infinitesimal generator associated with the diffusion

$$dX_t = F(X_t)dt + GdW_t.$$

The solution of this ergodic PDE is a couple (u, λ), where λ is a constant, which gives the optimal value for the ergodic control problem.

Then one can guess that the BSDE associated with this ergodic PDE should be

$$dY_t^x = -[\psi(X_t^x, Z_t^x) - \lambda]dt - Z_t^x dW_t,$$

where $t \in [0, +\infty)$.

The solution of this ergodic BSDE is a triple (Y^x, Z^x, λ), where λ is a constant, which gives the optimal value for the control problem.

The aim of this work is to give the first result about the wellposedness of this ergodic BSDE (existence, uniqueness and regularity), and we give also the link among ergodic control, ergodic PDE and ergodic BSDE. Once again, λ is the optimal value for the ergodic control problem.