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QUANTIZATION ON CURVED MANIFOLDS

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Abstract. Since the early days of quantum mechanics many techniques have been developed in order to deal with manifolds with non-trivial topology. Among them two techniques have received a great attention in the literature and are shortly reviewed here as they are most geometrical in nature. These are the Kostant–Souriau geometric quantization scheme and the so called constrained quantum mechanics. A notable difference between them are the geometrical structures used in these theories. The first is based on the symplectic structure of the phase space and the second one relies on the Riemannian metric of the configurational manifold. Both approaches are illustrated in full details. Presented examples include the n-dimensional variants of the harmonic oscillator and the Kepler problem which are treated within geometric quantization scheme by making use of the Marsden–Weinstein reduction theorem and even a combination of both methods is applied in the study of quantum-mechanical aspects of the geodesic flows on axisymmetric ellipsoids.

1. Introduction

The material presented in this report is based on a revised and expanded format of lectures delivered at the third edition of Varna International Conference on *Geometry, Integrability and Quantization* held in June 14–23, 2001.

No claims whatsoever are made with regard to completeness and at many places the reader is referred to the original works given in the list of references. The principal objective is to present the available techniques for treating quantum-mechanical systems defined on topologically non-trivial manifolds. As they appear even at early stages of quantum theory we start with a brief recapitulation of the old Bohr–Sommerfeld theory. Then we proceed with Kostant–Souriau