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ON THE REDUCTIONS AND HAMILTONIAN STRUCTURES OF N-WAVE TYPE EQUATIONS

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Abstract. The reductions of the integrable N-wave type equations solvable by the inverse scattering method with the generalized Zakharov–Shabat system L and related to some simple Lie algebra $\mathfrak{g}$ are analyzed. Special attention is paid to the $\mathbb{Z}_2$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$-reductions including ones that can be embedded also in the Weyl group of $\mathfrak{g}$. The consequences of these restrictions on the properties of their Hamiltonian structures are analyzed on specific examples which find applications to nonlinear optics.

1. Introduction

It is well known that the N-wave equations [1–6]

$$\mathrm{i}[J, Q_t] - \mathrm{i}[I, Q_x] + [[I, Q], [J, Q]] = 0\,, \tag{1}$$

are solvable by the inverse scattering method (ISM) [4, 5] applied to the generalized system of Zakharov–Shabat type [4, 7, 8]:

$$L(\lambda)\Psi(x,t,\lambda) = \left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x} + [J, Q(x,t)] - \lambda J\right)\Psi(x,t,\lambda) = 0, \quad J \in \mathfrak{h}\,, \tag{2}$$

$$Q(x,t) = \sum_{\alpha \in \Delta_+} (q_\alpha(x,t)E_\alpha + p_\alpha(x,t)E_{-\alpha}) \in \mathfrak{g}/\mathfrak{h}\,, \tag{3}$$

where $\mathfrak{h}$ is the Cartan subalgebra and E_α are the root vectors of the simple Lie algebra $\mathfrak{g}$. Indeed (1) can be written in the Lax form, or in other words, it is