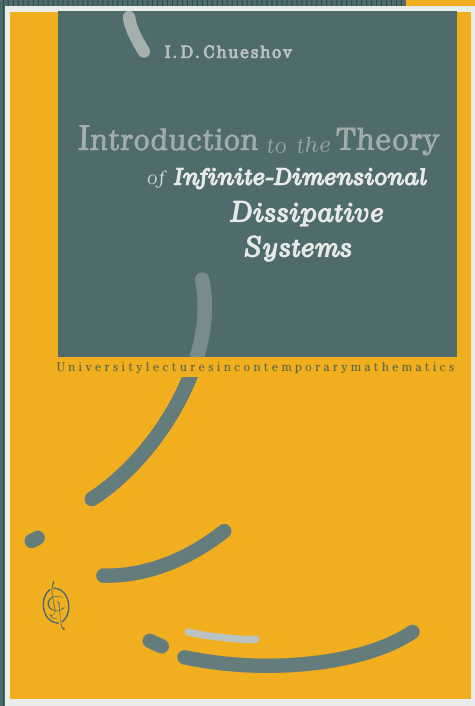


Author: I. D. Chueshov
Title: Introduction to the Theory
of Infinite-Dimensional
Dissipative Systems
ISBN: 966-7021-64-5



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This book provides an exhaustive introduction to the scope of main ideas and methods of the theory of infinite-dimensional dissipative dynamical systems which has been rapidly developing in recent years. In the examples systems generated by nonlinear partial differential equations arising in the different problems of modern mechanics of continua are considered. The main goal of the book is to help the reader to master the basic strategies used in the study of infinite-dimensional dissipative systems and to qualify him/her for an independent scientific research in the given branch. Experts in nonlinear dynamics will find many fundamental facts in the convenient and practical form in this book.

The core of the book is composed of the courses given by the author at the Department of Mechanics and Mathematics at Kharkov University during a number of years. This book contains a large number of exercises which make the main text more complete. It is sufficient to know the fundamentals of functional analysis and ordinary differential equations to read the book.

Translated by
Constantin I. Chueshov
from the Russian edition («ACTA», 1999)

Translation edited by
Maryna B. Khorolska

I. D. Chueshov

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ACTA 2002

UDC 517

2000 Mathematics Subject Classification:
primary 37L05; secondary 37L30, 37L25.

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Kharkiv, Ukraine
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Свідоцтво ДК №179

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ISBN 966-7021-20-3 (series)
ISBN 966-7021-64-5

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Палкой щупая дорогу,
Бродит наугад слепой,
Осторожно ставит ногу
И бормочет сам с собой.
И на бельмах у слепого
Полный мир отображен:
Дом, лужок, забор, корова,
Клочья неба голубого —
Все, чего не видит он.

Вл. Ходасевич
«Слепой»

A blind man tramps at random touching the road with a stick.
He places his foot carefully and mumbles to himself.
The whole world is displayed in his dead eyes.
There are a house, a lawn, a fence, a cow
and scraps of the blue sky — everything he cannot see.

Vl. Khodasevich
«A Blind Man»

Preface

The recent years have been marked out by an evergrowing interest in the research of qualitative behaviour of solutions to nonlinear evolutionary partial differential equations. Such equations mostly arise as mathematical models of processes that take place in real (physical, chemical, biological, etc.) systems whose states can be characterized by an infinite number of parameters in general. Dissipative systems form an important class of systems observed in reality. Their main feature is the presence of mechanisms of energy reallocation and dissipation. Interaction of these two mechanisms can lead to appearance of complicated limit regimes and structures in the system. Intense interest to the infinite-dimensional dissipative systems was significantly stimulated by attempts to find adequate mathematical models for the explanation of turbulence in liquids based on the notion of strange (irregular) attractor. By now significant progress in the study of dynamics of infinite-dimensional dissipative systems have been made. Moreover, the latest mathematical studies offer a more or less common line (strategy), which when followed can help to answer a number of principal questions about the properties of limit regimes arising in the system under consideration. Although the methods, ideas and concepts from finite-dimensional dynamical systems constitute the main source of this strategy, finite-dimensional approaches require serious reevaluation and adaptation.

The book is devoted to a systematic introduction to the scope of main ideas, methods and problems of the mathematical theory of infinite-dimensional dissipative dynamical systems. Main attention is paid to the systems that are generated by nonlinear partial differential equations arising in the modern mechanics of continua. The main goal of the book is to help the reader to master the basic strategies of the theory and to qualify him/her for an independent scientific research in the given branch. We also hope that experts in nonlinear dynamics will find the form many fundamental facts are presented in convenient and practical.

The core of the book is composed of the courses given by the author at the Department of Mechanics and Mathematics at Kharkov University during several years. The book consists of 6 chapters. Each chapter corresponds to a term course (34-36 hours) approximately. Its body can be inferred from the table of contents. Every chapter includes a separate list of references. The references do not claim to be full. The lists consist of the publications referred to in this book and offer additional works recommen-

ded for further reading. There are a lot of exercises in the book. They play a double role. On the one hand, proofs of some statements are presented as (or contain) cycles of exercises. On the other hand, some exercises contain an additional information on the object under consideration. We recommend that the exercises should be read at least. Formulae and statements have double indexing in each chapter (the first digit is a section number). When formulae and statements from another chapter are referred to, the number of the corresponding chapter is placed first.

It is sufficient to know the basic concepts and facts from functional analysis and ordinary differential equations to read the book. It is quite understandable for under-graduate students in Mathematics and Physics.

I.D. Chueshov