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GLOBAL EXISTENCE OF SOLUTION FOR REACTION DIFFUSION SYSTEMS WITH A FULL MATRIX OF DIFFUSION COEFFICIENTS

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Abstract. The goal of this work is to study the global existence in time of solutions for some coupled systems of reaction diffusion which describe the spread within a population of infectious disease. We consider a full matrix of diffusion coefficients and we show the global existence of the solutions.

1 Introduction

We are mainly interested in global existence in time of solutions to reaction-diffusion system of the form

$$\frac{\partial u}{\partial t} - a\Delta u - b\Delta v = \Pi - f(u,v) - \sigma u \quad \text{in} \quad]0,+\infty[\times \Omega \tag{1.1}$$

$$\frac{\partial v}{\partial t} - c\Delta u - a\Delta v = f(u,v) - \sigma v \quad \text{in} \quad]0,+\infty[\times \Omega \tag{1.2}$$

with the following boundary conditions

$$\frac{\partial u}{\partial \eta} = \frac{\partial v}{\partial \eta} = 0 \qquad \text{in} \qquad]0,+\infty[\times \partial\Omega \tag{1.3}$$

and the initial data

$$u(0,x) = u_0, \quad v(0,x) = v_0 \quad \text{in } \Omega. \tag{1.4}$$

where Ω is an open bounded domain in $\mathbb{R}^n$ with boundary $\partial\Omega$ of class C^1, $\frac{\partial}{\partial \eta}$ denotes the outwards normal derivative on $\partial\Omega$, Δ denotes the Laplacian operator with respect to the x variable, a, b, c, σ are positive constants, $c \geq 0$ satisfying the condition $(b+c) < 2a$ which reflects the parabolicity of the system, $\Pi \geq 0$.

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We assume that $b < c$,and the initial data are assumed to be in the following region

$$\Sigma = \left\{ (u_0, v_0) \in \mathbb{R}^2 \text{ such that } v_0 \geq \sqrt{\frac{c}{b}} \mid u_0 \mid \right\}.$$

For more details, one may consult [6].

The function f is nonnegative and continuously differentiable function on Σ such that

$$f(-\sqrt{\frac{b}{c}}\eta, \eta) = 0 \text{ and } f(\sqrt{\frac{b}{c}}\eta, \eta) \geq \frac{\Pi\sqrt{c}}{\sqrt{c} + \sqrt{b}}, \text{for all } \eta \geq 0. \tag{1.5}$$

In addition we suppose that

$$(\xi, \eta) \in \Sigma \Longrightarrow 0 \leq f(\xi, \eta) \leq \varphi(\xi)(1 + \eta)^{\beta}, \tag{1.6}$$

where $\beta \geq 1$ and φ is nonnegative function of class $C(\mathbb{R})$ such that

$$\lim_{\xi \to -\infty} \frac{\varphi(\xi)}{\xi} = 0. \tag{1.7}$$

B. Rebai [10] has proved the global existence of solutions for system (1.1)-(1.4), in the case $b = 0, c > 0$ (triangular matrix).The present investigation is a continuation of results obtained in [10].

In this study, we will treat the case of a general full matrix of diffusion coefficients satisfying $a = d$. Here, we make use of the Lyapunov function techniques and present an approach similar to that developed in [8] under the assumptions (1.6)-(1.7).

The components $u(t, x)$ and $v(t, x)$ represent either chemical concentrations or biological population densities and system (1.1)-(1.4) is a mathematical model describing various chemical and biological phenomena (see, e.g. Cussler [3]).

2 Local Existence and Invariant Regions

Throughout the text we shall denote by $\| \|_p$ the norm in $L^p(\Omega)$, $\| \|_\infty$the norm in $L^\infty(\Omega)$ or $C(\overline{\Omega})$.

For any initial data in $C(\overline{\Omega})$ or $L^p(\Omega)$, $p \in \,]1, +\infty[$, local existence and uniqueness of solutions to the initial value problem (1.1)-(1.4) follow from the basic existence theory for abstract semilinear differential equations (see D. Henry [5] and A. Pazy [9]). The solutions are classical on $]0; T^*[$, where T^* denotes the eventual blowing-up time in $L^\infty(\Omega)$.

Furthermore, if $T^* < +\infty$, then

$$\lim_{t\uparrow T^*}(\|u(t)\|_\infty + \|v(t)\|_\infty) = +\infty.$$

Therefore, if there exists a positive constant C such that

$$\|u(t)\|_\infty + \|v(t)\|_\infty \le C, \forall t \in \left]0, T^*\right[,$$

then $T^* = +\infty$.

Multiplying equation (1.1) through by $\sqrt{c}$ and equation (1.2) by $\sqrt{b}$, subtracting the resulting equations one time and adding them an other time we get

$$\frac{\partial w}{\partial t} - \left(a + \sqrt{bc}\right)\Delta w = \sqrt{c}\Pi - (\sqrt{c} - \sqrt{b})F(w, z) - \sigma w \quad \text{in } \left]0, T^*\right[\times \Omega, \tag{2.1}$$

$$\frac{\partial z}{\partial t} - \left(a - \sqrt{bc}\right)\Delta z = -\sqrt{c}\Pi + (\sqrt{c} + \sqrt{b})F(w, z) - \sigma z \text{ in } \left]0, T^*\right[\times \Omega, \tag{2.2}$$

with the boundary conditions

$$\frac{\partial w}{\partial \eta} = \frac{\partial z}{\partial \eta} = 0 \text{ in } \left]0, T^*\right[\times \partial\Omega, \tag{2.3}$$

and the initial data

$$w(0, x) = w_0(x), z(0, x) = z_0(x) \text{ in } \Omega, \tag{2.4}$$

where,

$$\begin{aligned} w(t, x) &= \sqrt{c}u(t, x) + \sqrt{b}v(t, x), \\ z(t, x) &= -\sqrt{c}u(t, x) + \sqrt{b}v(t, x), \end{aligned} \tag{2.5}$$

for any (t, x) in$\left]0, T^*\right[\times \Omega$ and

$$F(w, z) = f(u, v) \text{ for all } (u, v) \text{ in } \Sigma. \tag{2.6}$$

To prove that Σ is an invariant region for system (1.1)–(1.4) it suffices to prove that the region

$$\Sigma_1 = \left\{(w_0, z_0) \in \mathbb{R}^2 \text{ such that } w_0 \ge 0,\ z_0 \ge 0\right\}.$$

is invariant for system (2.1)–(2.4).

Now, to prove that the region Σ_1 is invariant for system (2.1)–(2.4), it suffices to show that $(\sqrt{c}\Pi - (\sqrt{c} - \sqrt{b})F(0, z)) \ge 0$ for $z \ge 0$ and $(-\sqrt{c}\Pi + (\sqrt{c} + \sqrt{b})F(w, 0)) \ge 0$, for $w \ge 0$, see [10].

From (1.5), its clear that the region $\Sigma_1$is invariant for system (2.1)–(2.4) and from (2.5) we have

$$\begin{aligned} u(t,x) &= \frac{1}{2\sqrt{c}}(w(t,x)-z(t,x)), \\ v(t,x) &= \frac{1}{2\sqrt{b}}(w(t,x)+z(t,x)). \end{aligned} \tag{2.7}$$

3 Existence of global solutions

By a simple application of comparison theorem [[10], Theorem 10.1] to system (2.1)-(2.4) implies that for any initial conditions $w_0 \geq 0$ and $z_0 \geq 0$, we have

$$0 \leq \ w(t,x) \leq \max(\|w_0\|_\infty, \frac{\sqrt{c}\Pi}{\sigma}) = K,$$

To prove the global existence of the solutions of problem (1.1)-(1.4),one needs to prove it for problem (2.1)-(2.4).To this subject, it is well known that,it suffices to derive a uniform estimate of the quantity $\left\|-\sqrt{c}\Pi+(\sqrt{c}+\sqrt{b})F(w,z)-\sigma z\right\|_p$ for some $p>\frac{n}{2}$, i.e.

$$\left\|-\sqrt{c}\Pi+(\sqrt{c}+\sqrt{b})F(w,z)-\sigma z\right\|_p \leq C,$$

where C is a nonnegative constant independent of t.

From the assumptions (1.6) and (1.7), we are led to establish the uniform boundedness of the $\|z\|_p$ on $]0,T^*[$ in order to get that of $\|z\|_\infty$ on $]0,T^*[$.

For $p \geq 2$, we put

$$\alpha = \frac{bc}{(a^2-bc)}, \alpha(p) = \frac{p\alpha+1}{p-1}, M_p = K + \frac{\sqrt{c}\Pi}{\sigma\alpha(p)}. \tag{3.1}$$

We firstly introduce the following lemmas, which are useful in our main results.

Lemma 1. *Let (w,z) be a solution of (2.1)-(2.4). Then*

$$\frac{d}{dt}\int_\Omega w dx + (\sqrt{c}-\sqrt{b})\int_\Omega F(w,z)\,dx + \sigma\int_\Omega w dx = \sqrt{c}\Pi\,|\Omega|\,. \tag{3.2}$$

Proof. We integrate both sides of (2.1) satisfied by w, which is positive and then we obtain

$$\frac{d}{dt}\int_\Omega w dx \;=\; \sqrt{c}\Pi\,|\Omega| - (\sqrt{c}-\sqrt{b})\int_\Omega F(w,z)\,dx - \sigma\int_\Omega w dx.$$

□

Lemma 2. *Assume that* $p \geq 2$ *and let*

$$G_q(t) = \int_\Omega \left[qw + \exp(-\frac{p-1}{p\alpha+1}\ln(\alpha(p)(M_p - w)))z^p \right] dt,$$

where (w, z) *is the solution of (2.1)-(2.4) on* $]0, T^*[$*. Then under the assumptions (1.6) -(1.7) there exist two positive constants* $q > 0$ *and* $s > 0$ *such that*

$$\frac{d}{dt} G_q(t) \leq -(p-1)\sigma G_q + s.$$

Proof. The proof is similar to that in Melkemi et al [8].

Let

$$h(w) = -\frac{p-1}{p\alpha+1}\ln(\alpha(p)(M_p - w)). \tag{3.3}$$

Then

$$G_q(t) = q\int_\Omega w dx + N(t), \tag{3.4}$$

where

$$N(t) = \int_\Omega e^{h(w)} z^p dx. \tag{3.5}$$

Differentiating $N(t)$ with respect to t and using the Green formula one obtains

$$\frac{d}{dt} N = H + S, \tag{3.6}$$

where

$$\begin{aligned} H &= -\left(a + \sqrt{bc}\right) \int_\Omega ((h'(w))^2 + h''(w)) e^{h(w)} z^p (\nabla w)^2 dx \\ &\quad -2pa \int_\Omega h'(w) e^{h(w)} z^{p-1} \nabla w \nabla z dx \\ &\quad -\left(a - \sqrt{bc}\right) \int_\Omega p(p-1) e^{h(w)} z^{p-2} (\nabla z)^2 dx, \end{aligned}$$

and

$$\begin{aligned} S &= \sqrt{c}\Pi \int_\Omega h'(w) e^{h(w)} z^p dx + \\ &\quad \int_\Omega \left[pz^{p-1}(\sqrt{c}+\sqrt{b})F(w,z) - (\sqrt{c}-\sqrt{b})h'(w)z^p F(w,z) \right] e^{h(w)} dx \\ &\quad -\sigma \int_\Omega h'(w) w e^{h(w)} z^p dx - -p\sigma \int_\Omega e^{h(w)} z^p dx - p\sqrt{c}\Pi \int_\Omega e^{h(w)} z^{p-1} dx. \end{aligned}$$

We observe that H is given by

$$H = -\int_\Omega Q e^{h(w)} dx,$$

where

$$Q = \left(a+\sqrt{bc}\right)((h'(w))^2+h''(w))z^p(\nabla w)^2+2pah'(w)z^{p-1}\nabla w\nabla z + \left(a-\sqrt{bc}\right)p(p-1)z^{p-2}(\nabla z)^2$$

is a quadratic form with respect to ∇w and ∇z, which is nonnegative if

$$(2pah'(w)z^{p-1})^2-4(a^2-bc)p(p-1)((h'(w))^2+h''(w))z^{2p-2}\leq 0. \tag{3.7}$$

We have chosen $h(w)$ such that

$$h\prime(w)=\frac{1}{\alpha(p)(M_p-w)},\ h''(w)=\frac{\alpha(p)}{(\alpha(p)(M_p-w))^2}.$$

It is easy to see that the left hand side of (3.7) can be written as

$$4(a^2-bc)pz^{2p-2}\left\{p\left[\alpha\frac{1}{(\alpha(p)(M_p-w))^2}-\frac{\alpha(p)}{(\alpha(p)(M_p-w))^2}\right]+\frac{1+\alpha(p)}{(\alpha(p)(M_p-w))^2}\right\}=0.$$

Since

$$p\alpha-p\alpha(p)+1+\alpha(p)=0,$$

the inequality (3.7) holds, $Q\geq 0$ and we have

$$H=-\int_\Omega Qe^{h(w)}dx\leq 0,$$

the second term S can be estimate as

$$\begin{aligned} S &\leq \int_\Omega(\sqrt{c}\Pi h'(w)-\sigma p)e^{h(w)}z^p dx + \\ &\quad \int_\Omega\left[pz^{p-1}(\sqrt{c}+\sqrt{b})F(w,z)-h'(w)z^p(\sqrt{c}-\sqrt{b}))F(w,z)\right]e^{h(w)}dx \\ &\leq -(p-1)\sigma\int_\Omega e^{h(w)}z^p dx + \\ &\quad \int_\Omega\left[(\sqrt{c}+\sqrt{b})pz^{p-1}F(w,z)-(\sqrt{c}-\sqrt{b})h'(w)z^pF(w,z)\right]e^{h(w)}dx, \end{aligned} \tag{3.8}$$

We have

$$h'(w)=\frac{1}{\alpha(p)(M_p-w)}\leq\frac{1}{\alpha(p)(M_p-K)}=\frac{\sigma}{\sqrt{c}\Pi}.$$

and

$$\begin{aligned} -h'(w) &= \frac{-1}{\alpha(p)(M_p-w)}\leq\frac{-1}{\alpha(p)M_p}, \\ h(w) &\leq \frac{-1}{\alpha(p)}\ln\frac{\sqrt{c}\Pi}{\sigma}. \end{aligned} \tag{3.9}$$

Taking into account the fact that $z \geq 0$, and from (3.9), we observe that

$$\begin{aligned} & pz^{p-1}(\sqrt{c}+\sqrt{b})F(w,z) - h'(w)z^p(\sqrt{c}-\sqrt{b})F(w,z) \\ \leq \ & (p(\sqrt{c}+\sqrt{b})z^{p-1} - \frac{1}{\alpha(p)M_p}(\sqrt{c}-\sqrt{b})z^p)F(w,z). \end{aligned}$$

Then for $\eta_0 = \frac{p(\sqrt{c}+\sqrt{b})}{(\sqrt{c}-\sqrt{b})}(\alpha(p)M_p+1) > 0$, and for $0 \leq \xi \leq K, \eta \geq \eta_0$, we have

$$\begin{aligned} & (p\eta^{p-1}(\sqrt{c}+\sqrt{b}) - \frac{1}{\alpha(p)M_p}(\sqrt{c}-\sqrt{b})\eta^p)F(\xi,\eta) \\ = & \left[\frac{p(\sqrt{c}+\sqrt{b})}{\eta} - \frac{(\sqrt{c}-\sqrt{b})}{\alpha(p)M_p}\right]\eta^p F(\xi,\eta) \leq 0, \end{aligned}$$

on the other hand, we deduce that the function $(\xi,\eta) \to p(\sqrt{c}+\sqrt{b})\eta^{p-1} - \frac{1}{\alpha(p))M_p}(\sqrt{c}-\sqrt{b})\eta^p$ is bounded on the compact interval $[0,\eta_0]$, then there exists $c_1 > 0$ such that

$$pz^{p-1}(\sqrt{c}+\sqrt{b})F(w,z) - (\sqrt{c}-\sqrt{b})h'(w)z^pF(w,z) \leq c_1F(w,z). \tag{3.10}$$

From (3.5), (3.8) and (3.10), we deduce immediately the following inequality

$$S \leq -(p-1)\sigma N + c_1\int_\Omega F(w,z)\,e^{h(w)}dx \leq -(p-1)\sigma N + c_1 e^{\frac{-1}{\alpha(p)}\ln\frac{\sqrt{c}\Pi}{\sigma}}\int_\Omega F(w,z)\,dx,$$

we put

$$q = \frac{c_1 e^{\frac{-1}{\alpha(p)}\ln\frac{\sqrt{c}\Pi}{\sigma}}}{(\sqrt{c}-\sqrt{b})},$$

by (3.2), we have

$$S \leq -(p-1)\sigma N + q\sqrt{c}\Pi\,|\Omega| - q\frac{d}{dt}\int_\Omega w(t,x)dx,$$

and from (3.4), it follows that

$$S \leq -(p-1)\sigma G_q + q((p-1)\sigma K + \sqrt{c}\Pi)\,|\Omega| - q\frac{d}{dt}\int_\Omega w(t,x)dx,$$

and from (3.4) and (3.6), we conclude that

$$\frac{d}{dt}G_q \leq -(p-1)\sigma G_q + s, \tag{3.11}$$

where

$$s = q((p-1)\sigma K + \sqrt{c}\Pi)\,|\Omega|.$$

□

Now we can establish the global existence and uniform boundedness of the solutions of (2.1)-(2.4).

Theorem 3. *Under the assumptions (1.6) and (1.7), the solutions of (2.1)-(2.4) are global and uniformly bounded on* $[0,+\infty[\times\Omega$.

Proof. Multiplying the inequality (3.11) by $e^{(p-1)\sigma t}$ and then integrating ,we deduce that there exists a positive constants $C>0$ independent of t such that:

$$G_q(t)\leq C.$$

From (3.3), we observe that

$$e^{h(w)}\geq e^{\frac{-1}{\alpha(p)}\ln(\alpha(p)M_p)},$$

it follows from (3.1) that for all $p\geq 2$,

$$\int_\Omega z^p dx\leq e^{\frac{1}{\alpha(p)}\ln(K\alpha(p)+\frac{\sqrt{c}\Pi}{\sigma})}G_q(t)\leq C_1(p),$$

where

$$C_1(p)=Ce^{\frac{1}{\alpha(p)}\ln(K\alpha(p)+\frac{\sqrt{c}\Pi}{\sigma})},$$

select $p>\frac{n}{2}$ and proceed to bounds $\left\|-\sqrt{c}\Pi+(\sqrt{c}+\sqrt{b})F(w,z)-\sigma z\right\|_p$.

Let

$$A=\max_{\xi_0\leq\xi\leq K_1}\varphi(\xi),$$

where

$$K_1=\frac{1}{2\sqrt{c}}K,$$

and ξ_0 is such that

$$\xi\leq\xi_0\Longrightarrow\varphi(\xi)\prec|\xi|,$$

since $\lim_{\xi\to-\infty}\frac{\varphi(\xi)}{\xi}=0\Longleftrightarrow\forall\varepsilon>0$, there exists ξ_0 such that for $\xi\leq\xi_0$,we have $\left|\frac{\varphi(\xi)}{\xi}\right|<\varepsilon$, using (1.6) and (2.6), we deduce that

$$F(w,z)=f(u,v)\leq\varphi(u)(1+v)^\beta,$$

which implies,

$$\begin{aligned}\int_\Omega F^p(w,z)\,dx &\leq \int_\Omega(\varphi(u))^p(1+v)^{\beta p})dx=\\ &\int_{u\leq\xi_0}(\varphi(u))^p(1+v)^{\beta p}dx+\int_{\xi_0\leq u}(\varphi(u))^p(1+v)^{\beta p}dx\\ &\leq \int_{u\leq\xi_0}|u|^p(1+v)^{\beta p}dx+A^p\int_{\xi_0\leq u}(1+v)^{\beta p}dx\end{aligned}$$

using (2.7), we have

$$|u|^p = \left|\frac{1}{2\sqrt{c}}(w(t,x) - z(t,x))\right|^p \leq (\frac{1}{2\sqrt{c}})^p(w(t,x) + z(t,x))^p,$$

then

$$\begin{aligned}
&\int_\Omega F^p\,(w,z)\,dx \\
\leq\ & \int_{u\leq\xi_0}(\frac{1}{2\sqrt{c}})^p(w+z)^p(1+\frac{1}{2\sqrt{b}}(w+z))^{\beta p})dx \\
&\qquad\qquad +A^p\int_{\xi_0\leq u}(1+\frac{1}{2\sqrt{b}}(w+z))^{\beta p}dx \\
\leq\ & \max(A^p,(\frac{1}{2\sqrt{c}})^p)(\int_{u\leq\xi_0}(w+z)^p(1+\frac{1}{2\sqrt{b}}(w+z))^{\beta p}dx \\
&\qquad\qquad +\int_{\xi_0\leq u}(1+\frac{1}{2\sqrt{b}}(w+z))^{\beta p}dx) \\
\leq\ & \max(A^p,(\frac{1}{2\sqrt{c}})^p)(\int_\Omega(w+z)^p(1+\frac{1}{2\sqrt{b}}(w+z))^{\beta p}dx \\
&\qquad\qquad +\int_\Omega(1+\frac{1}{2\sqrt{b}}(w+z))^{\beta p}dx).
\end{aligned}$$

We also have

$$\begin{aligned}
&\int_\Omega(w+z)^p(1+\frac{1}{2\sqrt{b}}(w+z))^{\beta p}dx \\
\leq\ & 2^{\beta p-1}(\int_\Omega(w+z)^p dx + (\frac{1}{2\sqrt{b}})^{\beta p}\int_\Omega(w+z)^{(\beta+1)p}dx) \\
\leq\ & 2^{(\beta+1)p-2}(K^P\,|\Omega| + C_1(p)) \\
&\qquad +2^{(2\beta+1)p-2}(\frac{1}{2\sqrt{b}})^{\beta p}(K^{(\beta+1)p}\,|\Omega| + C_1((\beta+1)p)) \\
=\ & C_2(\beta,p,K,\Omega),
\end{aligned}$$

and

$$\begin{aligned}
&\int_\Omega(1+\frac{1}{2\sqrt{b}}(w+z))^{\beta p}dx \leq \\
&\qquad 2^{\beta p-1}(|\Omega| + (\frac{1}{2\sqrt{b}})^{\beta p}\times 2^{\beta p-1}(K^{\beta p}\,|\Omega| + C_1(\beta p))) = C_3(\beta,p,K,\Omega)
\end{aligned}$$

Consequently,

$$\int_\Omega F^p\,(w,z)\,dx \leq C_4(A,\beta,p,K,\Omega).$$

Finally

$$\begin{aligned} & \left\| -\sqrt{c}\Pi + (\sqrt{c}+\sqrt{b})F(w,z) - \sigma z \right\|_p \\ \leq \ & (\sqrt{c}+\sqrt{b})\left\| F(w,z)\right\|_p + \sigma \left\| z\right\|_p + \sqrt{c}\Pi\left|\Omega\right| \\ \leq \ & (\sqrt{c}+\sqrt{b})\sqrt[p]{C_4(A,\beta,p,K)} + \sigma\sqrt[p]{C_1(p)} + \sqrt{c}\Pi\left|\Omega\right| \\ = \ & C_5(A,\beta,p,K,\Omega,\sigma). \end{aligned}$$

Using the regularity results for solutions of parabolic equations in [5], we conclude that the solutions of the problem (2.1)-(2.4) are uniformly bounded on $[0,+\infty[\times\Omega$.

By (2.7), its easy to see that the solutions of the problem (1.1)-(1.4) are also uniformly bounded on $[0,+\infty[\times\Omega$. □

Remark 4. *The condition of parabolicity implies that* $\det(A) = a^2 - bc > 0$,*where* A *is the matrix of diffusion.*

Remark 5. *Noting that if* $(\xi,\eta)\in\Sigma$, *then* $\xi\in\mathbb{R}$ *and* $\eta\geq 0$.

Remark 6. *Because* $0\leq\ w(t,x)\leq K$ *and* $z(t,x)\geq 0$, *we deduce that*

$$-\infty\ \leq u(t,x) = \frac{1}{2\sqrt{c}}(w(t,x)-z(t,x)) \leq \frac{1}{2\sqrt{c}}K = K_1.$$

References

[1] N. D. Alikakos, *L^p-Bounds of Solutions of Reaction-Diffusion Equations*, Comm. Partial. Differential. Equations **4** (1979), 827-868. MR0537465. Zbl 0421.35009.

[2] C. Castillo-Chavez, K. Cook, W. Huang and S. A. Levin, *On the role of long incubation periods in the dynamics of acquires immunodeficiency syndrome*, AIDS, J. Math. Biol. **27** (1989) 373-398. MR1009897. Zbl 0715.92029.

[3] E. L. Cussler, *Multicomponent diffusion*, Chemical Engineering Monographs **3** Elsevier Scientific Publishing Company, Amsterdam, 1976.

[4] A. Haraux and A.Youkana, *On a Result of K. Masuda Concerning Reaction-Diffusion Equations*, Tohoku. Math. J. **40** (1988), 159-163. MR0927083. Zbl 0689.35041.

[5] D. Henry, *Theory of Semi-Linear Parabolic Equations*, Lecture Notes in Math. **840**, Springer Verlas, New York, 1984.

[6] S. Kouachi, *Global Existence of solutions for reaction-diffusion systems with a full matrix of diffusion coefficients and non homogeneous boundary conditions*, Electron. J. Qual. Theory Differ. Equ. 2002, paper no. 2, 10pp. MR1884960. Zbl 0988.35078.

[7] K. Masuda, *On the Global Existence and Asymptotic Behavior of Solution of Reaction-Diffusion Equations*, Hokaido Math. J. **12** (1983), 360-370. MR0719974. Zbl 0529.35037.

[8] L. Melkemi, A. Z. Mokrane and A. Youkana, *On the uniform boundness of the solutions of systems of reaction-diffusion equtions*, Electron. J. Qual. Theory Differ. Equ. 2005, paper no. 24, 10pp. MR2191668. Zbl 1090.35095.

[9] A. Pazy, *Semigroups of linear operators and applications to partial differential equations*, Applied. Math. Sciences **44**, Springer-Verlag, New York, 1983. MR0710486. Zbl 0516.47023.

[10] B. Rebai, *Global classical solutions for Reaction-Diffusion Systems with a triangular Matrix of Diffusion Coefficients*, Electron. J. Differ. Equ. (EJDE) **2011** (2011), paper no. 99, 8pp. MR2832275. Zbl 1227.35151.

[11] J. Savchik, B. Changs and H. Rabitz, *Application of moments to the general linear multicomponent reaction-diffusion equations*, J. Phys. Chem. **37** (1983), 1990–1997.

[12] J. Smoller, *Shock Wavesand Reaction-Diffusion Equations*, Springer-Verlag, New York 1983. MR0688146. Zbl 0508.35002.

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