EXISTENCE RESULTS FOR NONLINEAR IMPLICIT FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we establish the existence and uniqueness of solution for a class of initial value problem for implicit fractional differential equations with Caputo fractional derivative. The arguments are based upon the Banach contraction principle, Schauder' fixed point theorem and the nonlinear alternative of Leray-Schauder type. As applications, two examples are included to show the applicability of our results.

Full text

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