

## EULER'S CONSTANT, SEQUENCES AND SOME ESTIMATES

Alina Sîntămărian

**Abstract.** We give a class of sequences with the argument of the logarithmic term modified and that converge quickly to a generalization of Euler's constant denoted by  $\gamma(a)$ , i.e. the limit of the sequence  $\left(\sum_{k=1}^n \frac{1}{a+k-1} - \ln \frac{a+n-1}{a}\right)_{n \in \mathbb{N}}$ , where  $a \in (0, +\infty)$ .

Also, we obtain estimates for  $\gamma - \left(\sum_{k=1}^n \frac{1}{k} - \ln \left(n + \frac{1}{2} + \frac{1}{24(n+1/2)}\right)\right)$ , where  $\gamma = \gamma(1)$  is the Euler's constant.

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Alina Sîntămărian  
Department of Mathematics,  
Technical University of Cluj-Napoca,  
Str. Memorandumului nr. 28,  
400114 Cluj-Napoca,  
Romania.  
e-mail: [Alina.Sintamarian@math.utcluj.ro](mailto:Alina.Sintamarian@math.utcluj.ro)

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