

A SHORT SURVEY OF THE DEVELOPMENT OF FIXED POINT THEORY

Santosh Kumar

Abstract. In this survey paper, we collected the developmental history of fixed point theory. Some important results from beginning up to now are incorporated in this paper.

[Full text](#)

References

- [1] Agarwal, R. P., Meehan, M. and O' Regan, D., *Fixed point theory and applications*, Cambridge University Press 2001. [Zbl 0960.54027](#). [MR 1825411](#).
- [2] Javid Ali and M. Imdad, *An implicit function implies several contraction conditions*, Sarajevo Journal of Mathematics **4** (17) (2008), 269-285. [Zbl 1180.54052](#). [MR 2483851](#).
- [3] I. Altun and D. Turkoglu, *Some fixed point theorems for weakly compatible mappings satisfying an implicit relation*, Taiwanese J.Math. **13**, **4** (2009), 1291-1301. [Zbl 1194.54055](#). [MR 2543744](#).
- [4] Andrzej, G. and Dugundji, A., *Fixed Point Theory*, Springer Verlag, New York (2003). [Zbl 1025.47002](#). [MR 0117588](#).
- [5] Banach, S., *Sur les operations dans les ensembles abstrits et leur applications aux équations intégrales*, Fund. Math. **3** (1922), 133 - 181. [JFM 48.1204.02](#).
- [6] V. Berinde, *Approximating fixed points of implicit almost contractions*, Hacettepe J.Math. Statistics, **41** (1)(2012), 93-102. [Zbl 06119892](#). [MR 2976915](#).
- [7] Border, K. C., *Fixed point theorems with applications to economics and game theory*, Cambridge University Press, 1985. [Zbl 0558.47038](#). [MR0790845](#).

2010 Mathematics Subject Classification: Primary 47H10; Secondary 54H26.

Keywords: Contraction map; Nonexpansive map; Fixed points; Iteration process; Densifying map; 1-set contraction; Approximation theory; Variational inequalities.

<http://www.utgjiu.ro/math/sma>

- [8] Brouwer, L. E. J., *Über Abbildung von Mannigfaltigkeiten*, Math. Ann. **71** (1912), 97 - 115. [JFM 42.0417.01](#). [MR 1511644](#).
- [9] Browder, F. E., *Fixed point theorems for noncompact mappings in Hilbert spaces*, Proc. Nat. Acad. Sci. USA **53** (1965), 1272 - 1276. [Zbl 0125.35801](#). [MR 0178324](#).
- [10] Browder, F. E., *Convergence of approximants to fixed point of nonexpansive nonlinear mappings in Banach spaces*, Arch. Rat. Mech. Anal. **21** (1967), 82 - 90. [Zbl 0148.13601](#). [MR 0206765](#).
- [11] Cheney, E. W. and Goldstein, A. A., *Proximity maps for convex sets*, Proc. Amer. Math. Soc. **10** (1959), 448 - 450. [Zbl 0092.11403](#). [MR 21#3755](#).
- [12] Fan, Ky, *A generalization of Tychonoff's fixed point theorem*, Math. Ann. **142** (1961), 305 - 310. [Zbl 0093.36701](#). [MR 0131268](#).
- [13] Fan, Ky, *Extensions of two fixed point theorems of F. E. Browder*, Math. Z. **112** (1969), 234 - 240. [Zbl 0185.39503](#). [MR 0251603](#).
- [14] Furi, M. and Vignoli, A., *Fixed point theorems in complete metric spaces*, Bull. Unione Mat Italiana **2** (1969), 505 - 509. [Zbl 0183.51404](#). [MR 0256378](#).
- [15] Gohde, D., *Zum prinzip der kontraktiven abbildung*, Math. Nachr. **30** (1965), 251 - 258. [Zbl 0127.08005](#). [MR 0190718](#).
- [16] Hartman, P. and Stampacchia, G., *On some nonlinear elliptic differential equations*, Acta Math. **115** (1966), 271 - 310. [Zbl 142.38102](#). [MR 34#6355](#).
- [17] M. Imdad, S. Kumar and M. S. Khan, *Remarks on some fixed point theorems satisfying implicit relations*, Radovi Matematicki, **11**(1) (2002), 135-143. [Zbl 1033.54025](#). [MR 1971330](#).
- [18] Ishikawa, S., *Fixed points by new iteration method*, Proc. Amer Math Soc. **44** (1974), 147 - 150. [Zbl 0286.47036](#). [MR 336469](#).
- [19] Kakutani, S., *A generalization of Brouwer fixed point theorem*, Duke Math. Journal, **8** (1941), 457 - 459. [Zbl 0061.40304](#). [MR 0004776](#).
- [20] Kirk, W. A., *A fixed point theorem for mappings which do not increase distances*, Amer. Math. Monthly, **72** (1965), 1004 - 1006. [Zbl 0141.32402](#). [MR 0189009](#).
- [21] Krasnosel'skii, M. A., *Two remarks on the method of successive approximations*, Uspekhi Mat. Nauk, **10: 1** (63) (1955), 123 - 127. [Zbl 0064.120002](#). [MR 0068119](#).
- [22] Mann, W. R., *Mean value methods in iteration*, Proc. Amer. Math. Soc. **4** (1953), 506- 510. [Zbl 0050.11603](#). [MR 0054846](#).

- [23] John F. Nash Jr., *Equilibrium points in n-person game*, Proceedings of the National Academy of Science, USA, **36** (1950), 48 - 49. Zbl 0036.01104. MR 0031701.
- [24] Noor, M. A., *An iterative algorithm for variational inequalities*, J. Math. Anal. Appl. **158** (1991), 448-455. Zbl 0733.65047. MR 1117574.
- [25] Nussbaum, R. D., *Some fixed point theorems*, Bull. Amer. Math. Soc. **77** (1971), 360 - 365. Zbl 0212.16502. MR 0284888.
- [26] Park, S., *Ninety years of the Brouwer fixed point theorem*, Vietnam J. Math. **27** (1999), 187 - 222. Zbl 938.54039. MR 2001k:55001.
- [27] Petryshyn, W. V., *Structure of fixed point sets of the k-set contractions*, Arch. Rat. Mech. Anal. **40** (1971), 312 - 328. Zbl 0218.47028. MR 273480.
- [28] V. Popa, *Fixed point theorems for implicit contractive mappings*, Stud. Cercet. Stiint. Ser. Mat. Univ. Bacau, **7** (1997), 127 - 133. Zbl 0967.54041. MR 1721711.
- [29] V. Popa, *Some fixed point theorems for compatible mappings satisfying an implicit relation*, Demonstratio Math. **32** (1) (1999), 157 - 163. Zbl 0926.54030. MR 1691726.
- [30] Rhoades, B. E., *Comments on two fixed point iteration methods*, J. Math. Anal. Appl. **56** (1976), 741 - 750. Zbl 0353.47029. MR 0430880.
- [31] E. Rothe, *Zur Theorie der topologischen Ordnung und der Vektorfelder in Banachschen Raumen*, Compositio Math. **5** (1937), 177-197. Zbl 0018.13304. MR 1556993.
- [32] I. A. Rus, *Generalized contractions and applications*, Cluj University Press, Cluj Napoca, 2001. Zbl 0968.54029. MR 1947742.
- [33] Sadovskii, B. N., *On a fixed point principle*, Funct. Anal. Appl., **1** (1967), 151-153. MR 211302.
- [34] Schauder, J., *Der Fixpunktsatz in Funktionalraumen*, Studia Math. **2** (1930), 171- 180. JFM 56.0355.01.
- [35] Singh, S. P. and Watson, B., *On approximating fixed points*, Proc. Symp. Pure Math. Amer. Math. Soc. Ed. F. E. Browder **45** (1986), 393- 395. Zbl 0597.47035. MR 843624.
- [36] Singh S. P., Watson. B. and Srivadtava, P., *Fixed Point Theory and Best Approximation: The KKM map Principle*, Kluwer Academic Publishers (1997) p. 220. Zbl 0901.47039. MR 1483076.

- [37] Tychonoff, A., *Ein Fixpunktsatz*, Math. Ann. **111** (1935), 767-776. [Zbl 0012.30803](#). [MR 1513031](#).
- [38] Zeidler, E., *Nonlinear functional analysis and applications I*, Springer Verlag, New York, 1985. [Zbl 0794.47033](#). [MR 0816732](#).

Santosh Kumar
Current affiliation:
Department of Mathematics
College of Natural and Applied Sciences
P.O.Box-35062
University of Dar es salaam
Tanzania.
e-mail: drsengr2002@gmail.com

On Leave from:
Department of Applied Mathematics
Inderprastha Engineering College,
Ghaziabad, U.P-201010.
India.

Surveys in Mathematics and its Applications **8** (2013), 91 – 101
<http://www.utgjiu.ro/math/sma>