

APPROXIMATE ANALYTICAL SOLUTION OF DIFFUSION EQUATION WITH FRACTIONAL TIME DERIVATIVE USING OPTIMAL HOMOTOPY ANALYSIS METHOD

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Abstract. In this article, optimal homotopy-analysis method is used to obtain approximate analytic solution of the time-fractional diffusion equation with a given initial condition. The fractional derivatives are considered in the Caputo sense. Unlike usual Homotopy analysis method, this method contains at the most three convergence control parameters which describe the faster convergence of the solution. Effects of parameters on the convergence of the approximate series solution by minimizing the averaged residual error with the proper choices of parameters are calculated numerically and presented through graphs and tables for different particular cases.

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