Surveys in Mathematics and its Applications

ISSN 1842-6298 (electronic), 1843-7265 (print)
Volume **6** (2011), 9 – 21

AN INTRODUCTION TO THE CHEEGER PROBLEM

Enea Parini

Abstract. Given a bounded domain $\Omega \subset \mathbb{R}^n$ with Lipschitz boundary, the Cheeger problem consists of finding a subset E of Ω such that its ratio perimeter/volume is minimal among all subsets of Ω. This article is a collection of some known results about the Cheeger problem which are spread in many classical and new papers.

Full text

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2010 Mathematics Subject Classification: 49Q20
Keywords: Cheeger problem.
The author acknowledges partial support from the DFG - Deutsche Forschungsgemeinschaft

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Enea Parini
Mathematisches Institut, Universität zu Köln
Weyertal 86-90
D-50931 Köln, Germany.
e-mail: eparini@math.uni-koeln.de