DYNAMIC SHORTFALL CONSTRAINTS FOR OPTIMAL PORTFOLIOS

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Abstract. We consider a portfolio problem when a Tail Conditional Expectation constraint is imposed. The financial market is composed of n risky assets driven by geometric Brownian motion and one risk-free asset. The Tail Conditional Expectation is calculated for short intervals of time and imposed as risk constraint dynamically. The method of Lagrange multipliers is combined with the Hamilton-Jacobi-Bellman equation to insert the constraint into the resolution framework. A numerical method is applied to obtain an approximate solution to the problem. We find that the imposition of the Tail Conditional Expectation constraint when risky assets evolve following a log-normal distribution, curbs investment in the risky assets and diverts the wealth to consumption.

Full text

References

[1] D. Akume, *Risk-Constrained Dynamic Portfolio Management*, PhD thesis, Department of Mathematics, University of Yaounde I, Cameroon, 2008.

[2] P. Artzner, F. Delbaen, J.-M. Eber and D. Heath, *Coherent Measures of Risk*, Mathematical Finance, **9** (3) (1999), 203-228. MR1850791(2002d:91056). Zbl 0980.91042.

[3] D. Cuoco, H. He, S. Isaenko, *Optimal Dynamic Trading Strategies with Risk Limits*, Operations Resaerch, **56** (2) (2008), 358-368. MR2410311. Zbl 1167.91365.

[4] W. H. Fleming and R. W. Rishel, *Deterministic and Stochastic Optimal Control*, Springer Verlag, Berlin, 1975. MR0454768(56#13016). Zbl 0323.49001.

[5] A. Gabih, W. Grecksch and R. Wunderlich, *Dynamic portfolio optimization with bounded shortfall risks*, Stochastic Analysis and Applications **23** (2005), 579–594. MR2140978(2006b:91069). Zbl 1121.91046.

[6] A. Gabih, J. Sass and R. Wunderlich, *Utility maximization under bounded expected loss*, Stochastic Models , **3** (25) (2009), 2009. MR2548057. Zbl pre05604648.

[7] R. Gerrard, S. Haberman and E. Vigna, *Optimal Investment Choices Post Retirement in a Defined Contribution Pension Scheme*, Insurance: Mathematics and Economics, **35** (2004), 321-342. MR2328631(2005g:91119). Zbl 1093.91027.

[8] H. He and N. Pearson, *Consumption and Portfolio Policies with Incomplete Markets and Short-Sale constraints: The Finite Dimensional Case*, Mathematical Finance, **1** (1991), 1-10. MR1122311(92j:90009). Zbl 0900.90142.

[9] E. Isaacson and H.B. Keller, *Analysis of Numerical Methods*, Dover Publications, New York, 1994. MR0201039(34#924). Zbl 0254.65001.

[10] P. Jorion, *Value at Risk: The New Benchmark for Managing Financial Risk*, McGraw-Hill Publishers, 1996.

[11] R. Korn, *Optimal Portfolios: Stochastic models for optimal investment and risk management in continuous time*, World Scientific, Singapore, 1997. MR1467390(98g:49021). Zbl 0931.91017.

[12] H. M. Markowitz, *Portfolio Selection*, The Journal of Finance, **7** (1) (1952), 77-91. MR2547483. Zbl 1173.01013.

[13] A. J. McNeil, R. Frey and P. Embrechts, *Quantitative Risk Management: Concepts, Techniques and Tools*, Princeton University Press, 2005. MR2175089(2006d:91005). Zbl 1089.91037.

[14] J. Nocedal and S. Wright, *Numerical Optimization*, Springer Verlag, Heidelberg, 2006. MR2244940(2007a:90001). Zbl 1104.65059.

[15] T. A. Pirvu, *Portfolio Optimization Under the Value-at-Risk Constraint*, Quantitative Finance **2** (7), (2007), 125-136. MR2325660(2008f:91119). Zbl 1180.91273.

[16] W. Putschögl and J. Sass, *Optimal Investment under Dynamic Risk Constraints and Partial Information*, Quantitative Finance, to appear.

[17] R. T. Rockafellar and S. Uryasev, *Conditional value-at-risk: optimization approach*, Dordrecht: Kluwer Academic Publishers. Appl. Optim., **54** (2001), 411-435. MR1835507(2002f:90073). Zbl 0989.91052.

[18] K. F. C. Yiu, *Optimal Portfolios under a Value-at-Risk Constraint*, Journal of Economic Dynamics and Control, **28** (2004), 1317-1334. MR2047993(2005f:91081). Zbl pre05362800.

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