

FIXED POINT THEOREMS FOR GENERALIZED WEAKLY CONTRACTIVE MAPPINGS

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Abstract. In this paper several fixed point theorems for generalized weakly contractive mappings in a metric space setting are proved. The set of generalized weakly contractive mappings considered in this paper contains the family of weakly contractive mappings as a proper subset. Fixed point theorems for single and multi-valued mappings, approximating scheme for common fixed point for some mappings, and fixed point theorems for fuzzy mappings are presented. It extends the work of several authors including Bose and Roychowdhury.

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