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On some 2-Banach spaces [1]

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Abstract

The main aim of this article is to introduce some difference sequence spaces with elements in a finite dimensional 2-normed space and extend the notion of 2-norm and derived norm to thus constructed spaces. We investigate the spaces under the action of different difference operators and show that these spaces become 2-Banach spaces when the base space is a 2-Banach space. We also prove that convergence and completeness in the 2-norm is equivalent to those in the derived norm as well as show that their topology can be fully described by using derived norm. Further we compute the 2-isometric spaces and prove the Fixed Point Theorem for these 2-Banach spaces.

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1 Introduction

The concept of 2-normed spaces was initially developed by Gähler [3] in the mid of 1960's. Since then, Gunawan and Mashadi [5], Gürdal [6] and many others have studied this concept and obtained various results.

Let X be a real vector space of dimension d, where $2 \leq d$. A real-valued function $\|.,.\|$ on X^2 satisfying the following four conditions:

(1) $\|x_1, x_2\| = 0$ if and only if x_1, x_2 are linearly dependent,

(2) $\|x_1, x_2\|$ is invariant under permutation,

(3) $\|\alpha x_1, x_2\| = |\alpha| \|x_1, x_2\|$, for any $\alpha \in R$,

(4) $\|x + x', x_2\| \leq \|x, x_2\| + \|x', x_2\|$

is called a 2-*norm* on X, and the pair $(X, \|.,.\|)$ is called a 2-normed space.

A sequence (x_k) in a 2-normed space $(X, \|.,.\|)$ is said to *converge* to some $L \in X$ in the 2-norm if

$$\lim_{k \to \infty} \|x_k - L, u_1\| = 0, \text{ for every } u_1 \in X.$$

A sequence (x_k) in a 2-normed space $(X, \|.,.\|)$ is said to be *Cauchy* with respect to the 2-norm if

$$\lim_{k,l \to \infty} \|x_k - x_l, u_1\| = 0, \text{ for every } u_1 \in X.$$

If every Cauchy sequence in X converges to some $L \in X$, then X is said to be complete with respect to the 2-norm. Any complete 2-normed space is said to be 2-Banach space.

The notion of difference sequence space was introduced by Kizmaz [7], who studied the difference sequence spaces $\ell_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$. The notion was further generalized by Et and Colak [1] by introducing the spaces $\ell_\infty(\Delta^s)$, $c(\Delta^s)$ and $c_0(\Delta^s)$. Another type of generalization of the difference sequence spaces is due to Tripathy and Esi [8], who studied the spaces $\ell_\infty(\Delta_m)$, $c(\Delta_m)$ and $c_0(\Delta_m)$. Tripathy, Esi and Tripathy [9] generalized the above notions and unified these as follows:

Let m, s be non-negative integers, then for Z a given sequence space we have

$$Z(\Delta_m^s) = \{x = (x_k) \in w : (\Delta_m^s x_k) \in Z\},$$

where $\Delta_m^s x = (\Delta_m^s x_k) = (\Delta_m^{s-1} x_k - \Delta_m^{s-1} x_{k+m})$ and $\Delta_m^0 x_k = x_k$ for all $k \in N$, which is equivalent to the following binomial representation:

$$\Delta_m^s x_k = \sum_{v=0}^{s} (-1)^v \binom{s}{v} x_{k+mv}.$$

Let m, s be non-negative integers, then for Z a given sequence space we define:

$$Z(\Delta_{(m)}^s) = \{x = (x_k) \in w : (\Delta_{(m)}^s x_k) \in Z\},$$

where $\Delta_{(m)}^s x = (\Delta_{(m)}^s x_k) = (\Delta_{(m)}^{s-1} x_k - \Delta_{(m)}^{s-1} x_{k-m})$ and $\Delta_{(m)}^0 x_k = x_k$ for all $k \in N$, which is equivalent to the following binomial representation:

$$\Delta_{(m)}^s x_k = \sum_{v=0}^{s} (-1)^v \binom{s}{v} x_{k-mv}.$$

It is important to note here that we take $x_{k-mv} = 0$, for non-positive values of $k - mv$.

Let $(X, \|.,.\|_X)$ be a finite dimensional real 2-normed space and $w(X)$ denotes X-valued sequence space. Then for non-negative integers m and s, we define the following sequence spaces:

$c_0(\|.,.\|, \Delta_{(m)}^s) = \{(x_k) \in w(X) : \lim_{k\to\infty} \|\Delta_{(m)}^s x_k, z_1\|_X = 0, \text{for every } z_1 \in X\}$,

$c(\|.,.\|, \Delta_{(m)}^s) = \{(x_k) \in w(X) : \lim_{k\to\infty} \|\Delta_{(m)}^s x_k - L, z_1\|_X = 0, \text{for some } L$ and for every $z_1 \in X\}$,

$\ell_\infty(\|.,.\|,\Delta^s_{(m)}) = \{(x_k) \in w(X) : \sup_k \|\Delta^s_{(m)}x_k, z_1\|_X < \infty, \text{for every } z_1 \in X\}$.

It is obvious that $c_0(\|.,.\|,\Delta^s_{(m)}) \subset c(\|.,.\|,\Delta^s_{(m)}) \subset \ell_\infty(\|.,.\|,\Delta^s_{(m)})$. Also for $Z = c_0, c$ and ℓ_∞, we have

$$Z(\|.,.\|,\Delta^i_{(m)}) \subset Z(\|.,.\|,\Delta^s_{(m)}), i = 0, 1, \ldots, s-1. \tag{1}$$

Similarly we can define the spaces $c_0(\|.,.\|,\Delta^s_m), c(\|.,.\|,\Delta^s_m)$ and $\ell_\infty(\|.,.\|,\Delta^s_m)$.

2 Discussions and Main Results

In this section we give some examples associated with 2-normed space and investigate the main results of this article involving the sequence spaces $Z(\|.,.\|,\Delta^s_{(m)})$ and $Z(\|.,.\|,\Delta^s_m)$, for $Z = c_0, c$ and ℓ_∞. Further we compute 2-isometric spaces and give the fixed point theorem for these spaces.

Example 1 *AS an example of a 2-normed space, we may take $X = R^2$ being equipped with the 2-norm $\|x, y\|$ = the area of the parallelogram spanned by the vectors x and y, which may be given explicitly by the formula:*

$$\|x, y\| = |x_1y_2 - x_2y_1|, x = (x_1, x_2), y = (y_1, y_2) \in X.$$

Example 2 *Let us take $X = R^2$ and consider a 2-norm $\|.,.\|_X$ as defined above. Consider the divergent sequence $x = \{\bar{1}, \bar{2}, \bar{3}, \ldots\} \in w(X)$, where $\bar{k} = (k, k)$, for each $k \in N$. But x belongs to $Z(\|.,.\|,\Delta)$ and $Z(\|.,.\|,\Delta_{(1)})$. Hence by (1) for every m, $s > 1$, x belong to $Z(\|.,.\|,\Delta^s_{(m)})$ and $Z(\|.,.\|,\Delta^s_m)$, for $Z = c_0, c$ and ℓ_∞.*

Theorem 1 *The spaces $Z(\|.,.\|,\Delta^s_{(m)})$ and $Z(\|.,.\|,\Delta^s_m)$, for $Z = c_0, c$ and ℓ_∞ are linear.*

Proof. Proof is easy and so omitted.

Theorem 2 *(i) Let Y be any one of the spaces $Z(\|.,.\|,\Delta^s_{(m)})$, for $Z=c_0,c$ and ℓ_∞. We define the following function $\|.,.\|_Y$ on $Y\times Y$ by*

$\|x,y\|_Y=0,$ *if x,y are linearly dependent,*

$= \sup_k \|\Delta^s_{(m)}x_k,z_1\|_X,$ *for every $z_1\in X$, if x,y are linearly independent.*

$$\text{Then } \|.,.\|_Y \text{ is a 2-norm on } Y. \tag{2}$$

(ii) Let H be any one of the spaces $Z(\|.,.\|,\Delta^s_m)$, for $Z=c_0,c$ and ℓ_∞. We define the following function $\|.,.\|_H$ on $H\times H$ by

$\|x,y\|_H=0,$ *if x,y are linearly dependent,*

$= \sum_{k=1}^{ms}\|x_k,z_1\|_X + \sup_k \|\Delta^s_m x_k,z_1\|_X,$ *for every $z_1\in X$, if x,y are linearly independent.*

$$\text{Then } \|.,.\|_H \text{ is a 2-norm on } Y. \tag{3}$$

Proof. (i) If x^1,x^2 are linearly dependent, then $\|x^1,x^2\|_Y=0$. Conversely assume $\|x^1,x^2\|_Y=0$. Then using (2), we have

$$\sup_k \|\Delta^s_{(m)}x^1_k,z_1\|_X=0, \text{ for every } z_1\in X.$$

This implies that

$$\|\Delta^s_{(m)}x^1_k,z_1\|=0, \text{ for every } z_1\in X \text{ and } k\geq 1.$$

Hence we must have

$$\Delta^s_{(m)}x^1_k=0 \text{ for all } k\geq 1.$$

Let $k=1$, then $\Delta^s_{(m)}x^1_1=\sum_{i=0}^{s}(-1)^i\binom{s}{v}x^1_{1-mi}=0$ and so $x^1_1=0$, by putting $x^i_{1-mi}=0$ for $i=1,\dots,s$. Similarly taking $k=2,\dots,ms$, we have $x^1_2=\dots=x^1_{ms}=0$. Next let $k=ms+1$, then $\Delta^s_{(m)}x^1_{ms+1}=\sum_{i=0}^{s}(-1)^i\binom{s}{v}x^1_{1+ms-mi}=0$.

Since $x^1_1 = x^1_2 = \cdots = x^1_{ms} = 0$, we have $x^1_{ms+1} = 0$. Proceeding in this way we can conclude that $x^1_k = 0$, for all $k \geq 1$. Hence $x^1 = \theta$ and so x^1, x^2 are linearly dependent.

It is obvious that $\|x^1, x^2\|_Y$ is invariant under permutation, since $\|x^2, x^1\|_Y = \sup_k \|z_1, \Delta^s_{(m)} x^1_k\|_X$ and $\|.,.\|_X$ is a 2-norm.

Let $\alpha \in R$ be any element. If $\alpha x^1, x^2$ are linearly dependent then it is obvious that

$$\|\alpha x^1, x^2\|_Y = |\alpha| \|x^1, x^2\|_Y.$$

Otherwise,

$$\|\alpha x^1, x^2\|_Y = \sup_k \|\Delta^s_{(m)} \alpha x^1_k, z_1\|_X = |\alpha| \sup_k \|\Delta^s_{(m)} x^1_k, z_1\|_X = |\alpha| \|x^1, x^2\|_Y.$$

Lastly, let $x^1 = (x^1_k)$ and $y^1 = (y^1_k) \in Y$. Then clearly

$$\|x^1 + y^1, x^2\|_Y \leq \|x^1, x^2\|_Y + \|y^1, x^2\|_Y.$$

Thus we can conclude that $\|.,.\|_Y$ is a 2-norm on Y.

(ii) For this part we shall only show that $\|x^1, x^2\|_H = 0$ implies x^1, x^2 are linearly dependent. Proof of other properties of 2-norm follow similarly with that of part (i).

Let us assume that $\|x^1, x^2\|_H = 0$. Then using (3), for every z_1 in X, we have

$$\sum_{k=1}^{ms} \|x^1_k, z_1\|_X + \sup_k \|\Delta^s_m x^1_k, z_1\|_X = 0 \qquad (4)$$

We have

$$\sum_{k=1}^{ms} \|x^1_k, z_1\|_X = 0, \text{ for every } z_1 \in X.$$

Hence

$$x_k^1 = 0, \text{ for } k = 1, 2, \ldots, ms.$$

Also we have from (4)

$$\sup_k \|\Delta_m^s x_k^1, z_1\|_X = 0 \text{ for every } z_1 \in X.$$

Hence we must have

$$\Delta_m^s x_k^1 = 0, \text{ for each } k \in N.$$

Let $k = 1$, then we have

$$\Delta_m^s x_1^1 = \sum_{v=0}^{s} (-1)^v \binom{s}{v} x_{1+mv}^1 = 0 \tag{5}$$

Also we have

$$x_k^1 = 0, \text{ for } k = 1 + mv, v = 1, 2, \ldots s - 1. \tag{6}$$

Thus from (5) and (6), we have $x_{1+ms}^1 = 0$. Proceeding in this way inductively, we have $x_k^1 = 0$, for each $k \in N$.
Hence $x^1 = \theta$ and so x^1, x^2 are linearly dependent.

Theorem 3 *Let Y be any one of the spaces $Z(\|.,.\|, \Delta_{(m)}^s)$, for $Z = c_0, c$ and ℓ_∞. We define the following function $\|.\|_\infty$ on Y by*
$\|x\|_\infty = 0,$ *if x is linearly dependent,*
$= \sup_k \max\{\|\Delta_{(m)}^s x_k, b_l\|_X : l = 1, \ldots, d\},$ *where $B = \{b_1, \ldots, b_d\}$ is a basis of X, if x is linearly independent.*

(7) *Then $\|.\|_\infty$ is a norm on Y and we call this as derived norm on Y.*

Proof. Proof is a routine verification and so omitted.

Remark 1 *Associated to the derived norm* $\|.\|_\infty$, *we can define balls(open)* $S(x,\varepsilon)$ *centered at* x *and radius* ε *as follows:*

$$S(x,\varepsilon) = \{y : \|x-y\|_\infty < \varepsilon\}.$$

Corollary 1 *The spaces* $Z(\|.,.\|,\Delta^s_{(m)})$, *for* $Z = c_0, c$ *and* ℓ_∞ *are normed linear spaces.*

Theorem 4 *If* X *is a 2-Banach space, then the spaces* $Z(\|.,.\|,\Delta^s_{(m)})$, *for* $Z = c_0, c$ *and* ℓ_∞ *are 2-Banach spaces under the 2-norm (2).*

Proof. We give the proof only for the space $\ell_\infty(\|.,.\|,\Delta^s_{(m)})$ and for other spaces it will follow on applying similar arguments.

Let (x^i) be any Cauchy sequence in $\ell_\infty(\|.,.\|,\Delta^s_{(m)})$ and $\varepsilon > 0$ be given. Then there exists a positive integer n_0 such that

$$\|x^i - x^j, u^1\|_Y < \varepsilon, \text{ for all } i,j \geq n_0 \text{ and for every } u^1.$$

Using the definition of 2-norm, we get

$$\sup_k \|\Delta^s_{(m)}(x^i_k - x^j_k), z_1\|_X < \varepsilon, \text{ for all } i,j \geq n_0 \text{ and for every } z_1 \in X.$$

It follows that

$$\|\Delta^s_{(m)}(x^i_k - x^j_k), z_1\|_X < \varepsilon, \text{ for all } i,j \geq n_0, k \in N \text{ and for every } z_1 \in X.$$

Hence $(\Delta^s_{(m)} x^i_k)$ is a Cauchy sequence in X for all $k \in N$ and so convergent in X for all $k \in N$, since X is a 2-Banach space. For simplicity, let

$$\lim_{i\to\infty} \Delta^s_{(m)} x^i_k = y_k, \text{ say, exists for each } k \in N.$$

Taking $k = 1, 2, \ldots, ms, \ldots$ we can easily conclude that

$$\lim_{i\to\infty} x^i_k = x_k, \text{ exists for each } k \in N.$$

Now for $i,j \geq n_0$, we have

$$\sup_k \|\Delta^s_{(m)}(x^i_k - x^j_k), z_1\|_X < \varepsilon, \text{ and for every } z_1 \in X.$$

Hence for every z_1 in X, we have

$$\sup_k \|\Delta^s_{(m)}(x^i_k - x_k), z_1\|_X < \varepsilon, \text{ for all } i \geq n_0 \text{ and as } j \to \infty.$$

It follows that $(x^i - x) \in \ell_\infty(\|.,.\|, \Delta^s_{(m)})$ and $\ell_\infty(\|.,.\|, \Delta^s_{(m)})$ is a linear space, so we have $x = x^i - (x^i - x) \in \ell_\infty(\|.,.\|, \Delta^s_{(m)})$. This completes the proof of the theorem.

Theorem 5 *Let Y be any one of the spaces $Z(\|.,.\|, \Delta^s_{(m)})$, for $Z = c_0, c$ and ℓ_∞. Then (x^i) converges to an x in Y in the 2-norm if and only if (x^i) also converges to x in the derived norm.*

Proof. Let (x^i) converges to x in Y in the 2-norm. Then

$$\|x^i - x, u^1\|_Y \to 0 \text{ as } i \to \infty \text{ for every } u^1.$$

Using (2), we get

$$\sup_k \|\Delta^s_{(m)}(x^i_k - x_k), z_1\|_X \to 0 \text{ as } i \to \infty \text{ for every } z_1 \in X.$$

Hence for any basis $\{b_1, b_2, \ldots, b_d\}$ of X, we have

$$\sup_k \max\{\|\Delta^s_{(m)}(x^i_k - x_k), b_l\|_X : l = 1,2,\ldots,d\} \to 0 \text{ as } i \to \infty.$$

Thus it follows that

$$\|x^i - x\|_\infty \to 0 \text{ as } i \to \infty.$$

Hence (x^i) converges to x in the derived norm.

Conversely assume (x^i) converges to x in the derived norm. Then we have

$$\|x^i - x\|_\infty \to 0 \text{ as } i \to \infty.$$

Hence using (7), we get

$$\sup_k \max\{\|\Delta^s_{(m)}(x^i_k - x_k), b_l\|_X : l = 1, 2, \ldots, d\} \to 0 \text{ as } i \to \infty.$$

Therefore

$$\sup_k \|\Delta^s_{(m)}(x^i_k - x_k), b_l\|_X \to 0 \text{ as } i \to \infty, \text{ for each } l = 1, \ldots, d.$$

Let y be any element of Y. Then

$$\|x^i - x, y\|_Y = \sup_k \|\Delta^s_{(m)}(x^i_k - x_k), z_l\|_X$$

Since $\{b_1, \ldots, b_d\}$ is a basis for X, z_1 can be written as

$$z_1 = \alpha_1 b_1 + \cdots + \alpha_d b_d \text{ for some } \alpha_1, \ldots, \alpha_d \in R.$$

Now

$$\|x^i - x, y\|_Y = \sup_k \|\Delta^s_{(m)}(x^i_k - x_k), z_l\|_X$$

$$\leq |\alpha_1| \sup_k \|\Delta^s_{(m)}(x^i_k - x_k), b_l\|_X + \cdots + |\alpha_d| \sup_k \|\Delta^s_{(m)}(x^i_k - x_k), b_d\|_X,$$

for each i in N.

Thus it follows that

$$\|x^i - x, y\|_Y \to 0 \text{ as } i \to \infty \text{ for every } y \in Y.$$

Hence (x^i) converges to x in Y in the 2-norm.

Corollary 2 *Let Y be any one of the spaces $Z(\|.,.\|, \Delta^s_{(m)})$, for $Z = c_0, c$ and ℓ_∞. Then Y is complete with respect to the 2-norm if and only if it is complete with respect to the derived norm.*

Summarizing remark 1, corollary 1 and corollary 2, we have the following result:

Theorem 6 *The spaces $Z(\|.,.\|,\Delta^s_{(m)})$, for $Z = c_0, c$ and ℓ_∞ are normed spaces and their topology agree with that generated by the derived norm $\|.\|_\infty$.*

Remark 2 *We get similar results as those of Theorem 3, Corollary 1, Theorem 4, Theorem 5, Corollary 2 and Theorem 6 for the spaces $Z(\|.,.\|,\Delta^s_m)$, for $Z = c_0, c$ and ℓ_∞ also.*

A 2-norm $\|.,.\|_1$ on a vector space X is said to be equivalent to a 2-norm $\|.,.\|_2$ on X if there are positive numbers A and B such that for all $x, y \in X$ we have

$$A\|x,y\|_2 \leq \|x,y\|_1 \leq B\|x,y\|_2.$$

This concept is motivated by the fact that equivalent norms on X define the same topology for X.

Remark 3 *It is obvious that any sequence $x \in Z(\|.,.\|,\Delta^s_{(m)})$ if and only if $x \in Z(\|.,.\|,\Delta^s_m)$, for $Z = c_0, c$ and ℓ_∞. Also it is clear that the two 2-norms $\|.,.\|_Y$ and $\|.,.\|_H$ defined by (2)and (3) are equivalent.*

Let X and Y be linear 2-normed spaces and $f : X \to Y$ a mapping. We call f an 2-isometry if

$$\|x_1 - y_1, x_2 - y_2\| = \|f(x_1) - f(y_1), f(x_2) - f(y_2)\|,$$

for all $x_1, x_2, y_1, y_2 \in X$.

Theorem 7 *For $Z = c_0, c$ and ℓ_∞, the spaces $Z(\|.,.\|,\Delta^s_{(m)})$ and $Z(\|.,.\|,\Delta^s_m)$ are 2-isometric with the spaces $Z(\|.,.\|)$.*

Proof. Let us consider the mapping

$$F : Z(\|.,.\|,\Delta^s_{(m)}) \to Z(\|.,.\|), \text{ defined by}$$

$$Fx = y = (\Delta^s_{(m)} x_k), \text{ for each } x = (x_k) \in Z(\|.,.\|, \Delta^s_{(m)}).$$

Then clearly F is linear. Since F is linear, to show F is a 2-isometry, it is enough to show that

$$\|F(x^1), F(x^2)\|_1 = \|x^1, x^2\|_Y, \text{ for every } x^1, x^2 \in Z(\|.,.\|, \Delta^s_{(m)}).$$

Now using the definition of 2-norm (2), without loss of generality we can write

$$\|x^1, x^2\|_Y = \sup_k \|\Delta^s_{(m)} x^1_k, z_1\|_X = \|F(x^1), F(x^2)\|_1,$$

where $\|.,.\|_1$ is a 2-norm on $Z(\|.,.\|)$, which can be obtained from (2) by taking $s = 0$.

In view of remark 3, we can define same mapping on the spaces $Z(\|.,.\|, \Delta^s_m)$ and completes the proof.

For the next Theorem let Y to be any one of the spaces $Z(\|.,.\|, \Delta^s_{(m)})$, for Z $= c_0, c$ and ℓ_∞.

Theorem 8 (**Fixed Point Theorem**) *Let Y be a 2-Banach space under the 2-norm (2), and T be a contractive mapping of Y into itself, that is, there exists a constant $C \in (0,1)$ such that*

$$\|Ty^1 - Tz^1, x^2\|_Y \leq C\|y^1 - z^1, x^2\|_Y,$$

for all y^1, z^1, x^2 in Y. Then T has a unique fixed point in Y.

Proof. If we can show that T is also contractive with respect to derived norm, then we are done by corollary 2 and the fixed point theorem for Banach spaces.

Now by hypothesis

$$\|Ty^1 - Tz^1, x^2\|_Y \leq C\|y^1 - z^1, x^2\|_Y, \text{ for all } y^1, z^1, x^2 \in Y.$$

This implies that

$$\sup_k \|\Delta^s_{(m)}(Ty^1_k - Tz^1_k), u_1\|_X \leq C \sup_k \|\Delta^s_{(m)}(y^1_k - z^1_k), u_1\|_X, \text{ for every } u_1 \in X.$$

Then for a basis $\{e_1, \ldots, e_d\}$ of X, we get

$$\sup_k \|\Delta^s_{(m)}(Ty^1_k - Tz^1_k), e_i\|_X \leq C \sup_k \|\Delta^s_{(m)}(y^1_k - z^1_k), e_i\|_X,$$

for all y^1, z^1 in Y and $i = 1, \ldots, d$.

Thus

$$\|Ty^1_k - Tz^1_k\|_\infty \leq C\|y^1_k - z^1_k\|_\infty.$$

That is T is contractive with respect to derived norm. This completes the proof.

Remark 4 *We get the fixed point theorem for the spaces* $Z(\|.,.\|, \Delta^s_m)$, *for* $Z = c_0, c$ *and* ℓ_∞ *as above.*

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