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COEFFICIENT ESTIMATES FOR TWO NEW SUBCLASSES OF BI-UNIVALENT FUNCTIONS

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Abstract. In the present investigation, we introduce and investigate two new subclasses of the function Σ of bi-univalent functions defined in the open unit disc. We find estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in the function classes $\beta_{\Sigma}(h, \lambda, \mu)$ and $B_{\Sigma}(n, h, \lambda)$. The results presented in this paper improve or generalize the recent works of *Keerthi* and *Raja* [14] and Porwal and Darus [20].

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1. Introduction and Definitions

Let A denote the class of analytic functions in the unit disk

$$U = \{z \in \mathbb{C} : |z| < 1\}$$

that have the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

and let S be the class of all functions from A which are univalent in U.

The Koebe one-quarter theorem [9] states that the image of U under every function f from S contains a disk of radius $\frac{1}{4}$. Thus every such univalent function has an inverse f^{-1} which satisfies

$$f^{-1}(f(z)) = z \ , \ \ (z \in U)$$

and

$$f\left(f^{-1}(w)\right) = w \ , \ \ \left(|w| < r_0(f) \ , \ r_0(f) \geq \frac{1}{4}\right),$$

where

$$f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots .$$

A function $f(z) \in A$ is said to be bi-univalent in U if both $f(z)$ and $f^{-1}(z)$ are univalent in U. Let Σ denote the class of bi-univalent functions defined in the unit disk U. For a brief history and interesting examples of functions in the class Σ; see [4].

The research into Σ was started by Lewin ([16]).It focused on problems connected with coefficients and obtained the bound for the second coefficient. Several authors have subsequently studied similar problems in this direction (see [5], [19]). Recently, Srivastava et al. [22] introduced and investigated subclasses of the bi-univalent functions and obtained bounds for the initial coefficients; it was followed by such works as those by Murugunsundaramoorthy et al. [18], Frasin and Aouf [10], Çağlar et al. [7] and others (see, for example, [1, 3, 8, 14, 15, 17, 20, 24],).

Not much is known about the bounds on the general coefficient $|a_n|$ for $n \geq 4$. In the literature, the only a few works determining the general coefficient bounds $|a_n|$ for the analytic bi-univalent functions ([2, 6, 11, 12, 13]). The coefficient estimate problem for each of $|a_n|$ ($n \in \mathbb{N} \backslash \{1, 2\}$; $\mathbb{N} = \{1, 2, 3, ...\}$) is still an open problem.

Definition 1. *Let the functions $h, p : U \to \mathbb{C}$ be so constrained that*

$$min\{Re(h(z)), Re(p(z))\} > 0$$

and

$$h(0) = p(0) = 1.$$

Definition 2. *A function $f \in \Sigma$ is said to be in the class $\beta_\Sigma(h, \lambda, \mu)$, $0 \leq \mu \leq \lambda \leq 1$, if the following conditions are satisfied:*

$$\frac{\lambda\mu z^3 f'''(z) + (2\lambda\mu + \lambda - \mu) z^2 f''(z) + z f'(z)}{\lambda\mu z^2 f''(z) + (\lambda - \mu) z f'(z) + (1 - \lambda + \mu) f(z)} \in h(U) \quad (z \in U) \tag{2}$$

and

$$\frac{\lambda\mu w^3 g'''(w) + (2\lambda\mu + \lambda - \mu) w^2 g''(w) + w g'(w)}{\lambda\mu w^2 g''(w) + (\lambda - \mu) w g'(w) + (1 - \lambda + \mu) g(w)} \in p(U) \quad (w \in U) \tag{3}$$

where $g(w) = f^{-1}(w)$.

Definition 3. *A function $f \in \Sigma$ is said to be $B_\Sigma(n, h, \lambda)$, $n \in \mathbb{N}_0$ and $\lambda \geq 1$, if the following conditions are satisfied:*

$$\frac{(1 - \lambda) D^n f(z) + \lambda D^{n+1} f(z)}{z} \in h(U) \quad (z \in U) \tag{4}$$

and

$$\frac{(1-\lambda)\,D^{n}g\,(w)+\lambda D^{n+1}g\,(w)}{w}\in p\,(U)\qquad (w\in U) \tag{5}$$

where $g\,(w)=f^{-1}\,(w)$ *and* D^{n} *stands for Salagean derivative introduced by Salagean [21].*

2. Coefficient Estimates

Theorem 1. *Let* f *given by (1) be in the class* $\beta_{\Sigma}\,(h,\lambda,\mu)$. *Then*

$$|a_2|\leq \min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{2\,(1+\lambda-\mu+2\lambda\mu)^2}},\sqrt{\frac{|h''(0)|+|p''(0)|}{4\left[(2+4\lambda-4\mu+12\lambda\mu)-(1+\lambda-\mu+2\lambda\mu)^2\right]}}\right\} \tag{6}$$

and

$$|a_3|\leq \min\left\{\begin{array}{l}\frac{|h'(0)|^2+|p'(0)|^2}{2(1+\lambda-\mu+2\lambda\mu)^2}+\frac{|h''(0)|+|p''(0)|}{8(1+2\lambda-2\mu+6\lambda\mu)},\\[2mm] \qquad\frac{\left[2(2+4\lambda-4\mu+12\lambda\mu)-(1+\lambda-\mu+2\lambda\mu)^2\right]|h''(0)|+(1+\lambda-\mu+2\lambda\mu)^2|p''(0)|}{8(1+2\lambda-2\mu+6\lambda\mu)\left[(2+4\lambda-4\mu+12\lambda\mu)-(1+\lambda-\mu+2\lambda\mu)^2\right]}\end{array}\right\}. \tag{7}$$

Proof. Let $f\in\beta_{\Sigma}\,(h,\lambda,\mu)$ and $0\leq\mu\leq\lambda\leq 1$. It follows from (2) and (3) that

$$\frac{\lambda\mu z^3 f'''(z)+(2\lambda\mu+\lambda-\mu)\,z^2 f''(z)+zf'(z)}{\lambda\mu z^2 f''(z)+(\lambda-\mu)\,zf'(z)+(1-\lambda+\mu)\,f(z)}=h\,(z) \tag{8}$$

and

$$\frac{\lambda\mu w^3 g'''(w)+(2\lambda\mu+\lambda-\mu)\,w^2 g''(w)+wg'(w)}{\lambda\mu w^2 g''(w)+(\lambda-\mu)\,wg'(w)+(1-\lambda+\mu)\,g(w)}=p\,(w)\,, \tag{9}$$

where $h\,(z)$ and $p\,(w)$ satisfy the conditions of Definiton 1. Furthermore, the functions $h\,(z)$ and $p\,(w)$ have the following Taylor-Maclaurin series expansions:

$$h\,(z)=1+h_1z+h_2z^2+\cdots$$

and

$$p\,(w)=1+p_1w+p_2w^2+\cdots,$$

respectively. Since

$$\frac{\lambda\mu z^3 f'''(z)+(2\lambda\mu+\lambda-\mu)\,z^2 f''(z)+zf'(z)}{\lambda\mu z^2 f''(z)+(\lambda-\mu)\,zf'(z)+(1-\lambda+\mu)\,f(z)}=1+(1+\lambda-\mu+2\lambda\mu)\,a_2z$$

$$+\left[2\,(1+2\lambda-2\mu+6\lambda\mu)\,a_3-(1+\lambda-\mu+2\lambda\mu)^2\,a_2^2\right]z^2+\cdots$$

and

$$\frac{\lambda\mu w^3 g'''(w) + (2\lambda\mu + \lambda - \mu) w^2 g''(w) + w g'(w)}{\lambda\mu w^2 g''(w) + (\lambda - \mu) w g'(w) + (1 - \lambda + \mu) g(w)} = 1 - (1 + \lambda - \mu + 2\lambda\mu) a_2 w$$
$$+ \left[2(1 + 2\lambda - 2\mu + 6\lambda\mu)\left(2a_2^2 - a_3\right) - (1 + \lambda - \mu + 2\lambda\mu)^2 a_2^2\right] w^2 + \cdots,$$

it follows from (8) and (9) that

$$(1 + \lambda - \mu + 2\lambda\mu) a_2 = h_1, \tag{10}$$

$$2(1 + 2\lambda - 2\mu + 6\lambda\mu) a_3 - (1 + \lambda - \mu + 2\lambda\mu)^2 a_2^2 = h_2, \tag{11}$$

and

$$-(1 + \lambda - \mu + 2\lambda\mu) a_2 = p_1, \tag{12}$$

$$2(1 + 2\lambda - 2\mu + 6\lambda\mu)\left(2a_2^2 - a_3\right) - (1 + \lambda - \mu + 2\lambda\mu)^2 a_2^2 = p_2. \tag{13}$$

From (10) and (12) we obtain

$$h_1 = -p_1,$$

and

$$2(1 + \lambda - \mu + 2\lambda\mu)^2 a_2^2 = h_1^2 + p_1^2. \tag{14}$$

By adding (11) to (13), we find that

$$\left[4(1 + 2\lambda - 2\mu + 6\lambda\mu) - 2(1 + \lambda - \mu + 2\lambda\mu)^2\right] a_2^2 = h_2 + p_2, \tag{15}$$

which gives us the desired estimate on $|a_2|$ as asserted in (6).

Next, in order to find the bound on $|a_3|$, by subtracting (13) from (11), we obtain

$$4(1 + 2\lambda - 2\mu + 6\lambda\mu) a_3 - 4(1 + 2\lambda - 2\mu + 6\lambda\mu) a_2^2 = h_2 - p_2. \tag{16}$$

Then, in view of (14) and (15) , it follows that

$$a_3 = \frac{h_1^2 + p_1^2}{2(1 + \lambda - \mu + 2\lambda\mu)^2} + \frac{h_2 - p_2}{4(1 + 2\lambda - 2\mu + 6\lambda\mu)}$$

and

$$a_3 = \frac{h_2 + p_2}{4(1 + 2\lambda - 2\mu + 6\lambda\mu) - 2(1 + \lambda - \mu + 2\lambda\mu)^2} + \frac{h_2 - p_2}{4(1 + 2\lambda - 2\mu + 6\lambda\mu)}.$$

as claimed. This completes the proof of Theorem 1.

Theorem 2. *Let f given by (1) be in the class $B_\Sigma(n,h,\lambda)$, $n \in \mathbb{N}_0$, $\lambda \geq 1$. Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2 + |p'(0)|^2}{(1+\lambda)^2 2^{2n+1}}}, \sqrt{\frac{|h''(0)| + |p''(0)|}{4(1+2\lambda)3^n}}\right\} \tag{17}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2 + |p'(0)|^2}{(1+\lambda)^2 2^{2n+1}} + \frac{|h''(0)| + |p''(0)|}{4(1+2\lambda)3^n}, \frac{|h''(0)|}{2(1+2\lambda)3^n}\right\}. \tag{18}$$

Proof. Let $f \in B_\Sigma(n,h,\lambda)$, $n \in \mathbb{N}_0, \lambda \geq 1$. It follows from (4) and (5) that

$$\frac{(1-\lambda)D^n f(z) + \lambda D^{n+1} f(z)}{z} = h(z) \tag{19}$$

and

$$\frac{(1-\lambda)D^n g(w) + \lambda D^{n+1} g(w)}{w} = p(w), \tag{20}$$

where $h(z)$ and $p(w)$ satisfy the conditions of Definiton 1.

It follows from (19) and (20) that

$$\left[(1-\lambda)2^n + \lambda 2^{n+1}\right] a_2 = h_1, \tag{21}$$

$$\left[(1-\lambda)3^n + \lambda 3^{n+1}\right] a_3 = h_2, \tag{22}$$

and

$$-\left[(1-\lambda)2^n + \lambda 2^{n+1}\right] a_2 = p_1, \tag{23}$$

$$\left[(1-\lambda)3^n + \lambda 3^{n+1}\right]\left(2a_2^2 - a_3\right) = p_2. \tag{24}$$

From (21) and (23) we obtain

$$h_1 = -p_1,$$

and

$$(1+\lambda)^2 2^{2n+1} a_2^2 = h_1^2 + p_1^2. \tag{25}$$

By adding (24) to (22), we find that

$$2(1+2\lambda)3^n a_2^2 = h_2 + p_2, \tag{26}$$

which gives us the desired estimate on $|a_2|$ as asserted in (17).

Next, in order to find the bound on $|a_3|$, by subtracting (24) from (22), we obtain

$$2(1+2\lambda)3^n a_3 - 2(1+2\lambda)3^n a_2^2 = h_2 - p_2.$$

Then, in view of (25) and (26) , it follows that

$$a_3 = \frac{h_1^2 + p_1^2}{(1+\lambda)^2 \, 2^{2n+1}} + \frac{h_2 - p_2}{2\,(1+2\lambda)\,3^n}$$

and

$$a_3 = \frac{h_2 + p_2}{2\,(1+2\lambda)\,3^n} + \frac{h_2 - p_2}{2\,(1+2\lambda)\,3^n}.$$

as claimed. This completes the proof of Theorem 2.

3. Corollaries and Consequences

Corollary 3. *If let*

$$h(z) = p(z) = \left(\frac{1+z}{1-z}\right)^{\alpha} = 1 + 2\alpha z + 2\alpha^2 z^2 + ... \qquad (0 < \alpha \leq 1),$$

then inequalities (6) and (7) become

$$|a_2| \leq \min\left\{\frac{2\alpha}{1+\lambda-\mu+2\lambda\mu}, \sqrt{\frac{2}{(2+4\lambda-4\mu+12\lambda\mu)-(1+\lambda-\mu+2\lambda\mu)^2}}\alpha\right\}$$

and

$$|a_3| \leq \min\left\{\frac{4\alpha^2}{(1+\lambda-\mu+2\lambda\mu)^2} + \frac{\alpha^2}{1+2\lambda-2\mu+6\lambda\mu}, \frac{2\alpha^2}{(2+4\lambda-4\mu+12\lambda\mu)-(1+\lambda-\mu+2\lambda\mu)^2}\right\}.$$

Corollary 4. *If let*

$$h(z) = p(z) = \frac{1+(1-2\beta)z}{1-z} = 1 + 2(1-\beta)z + 2(1-\beta)z^2 + \cdots \qquad (0 \leq \beta < 1),$$

then inequalities (6) and (7) become

$$|a_2| \leq \min\left\{\frac{2(1-\beta)}{1+\lambda-\mu+2\lambda\mu}, \sqrt{\frac{2(1-\beta)}{(2+4\lambda-4\mu+12\lambda\mu)-(1+\lambda-\mu+2\lambda\mu)^2}}\right\}.$$

and

$$|a_3| \leq \min\left\{\frac{4(1-\beta)^2}{(1+\lambda-\mu+2\lambda\mu)^2} + \frac{1-\beta}{1+2\lambda-2\mu+6\lambda\mu}, \frac{2(1-\beta)}{(2+4\lambda-4\mu+12\lambda\mu)-(1+\lambda-\mu+2\lambda\mu)^2}\right\}.$$

Remark 1. *Corollary 3 and Corollary 4 provide an improvement estimates obtained by Keerthi and Raja [14].*

Taking $\mu = 0$ in Theorem 1, we get

Corollary 5. *If* $f \in \beta_{\Sigma}(h, \lambda)$ *then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2 + |p'(0)|^2}{2(1+\lambda)^2}}, \sqrt{\frac{|h''(0)| + |p''(0)|}{4(1+2\lambda-\lambda^2)}}\right\} \tag{27}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2 + |p'(0)|^2}{2(1+\lambda)^2} + \frac{|h''(0)| + |p''(0)|}{8(1+2\lambda)}, \frac{(3+6\lambda-\lambda^2)|h''(0)| + (1+\lambda)^2 |p''(0)|}{8(1+2\lambda)(1+2\lambda-\lambda^2)}\right\} \tag{28}$$

Corollary 6. *If let*

$$h(z) = p(z) = \left(\frac{1+z}{1-z}\right)^{\alpha} = 1 + 2\alpha z + 2\alpha^2 z^2 + ... \qquad (0 < \alpha \leq 1),$$

then inequalities (27) and (28) become

$$|a_2| \leq \min\left\{\frac{2\alpha}{1+\lambda}, \sqrt{\frac{2}{1+2\lambda-\lambda^2}}\alpha\right\}$$

and

$$|a_3| \leq \min\left\{\frac{4\alpha^2}{(1+\lambda)^2} + \frac{\alpha^2}{1+2\lambda}, \frac{2\alpha^2}{1+2\lambda-\lambda^2}\right\}.$$

Corollary 7. *If let*

$$h(z) = p(z) = \frac{1+(1-2\beta)z}{1-z} = 1 + 2(1-\beta)z + 2(1-\beta)z^2 + \cdots \qquad (0 \leq \beta < 1),$$

then inequalities (27) and (28) become

$$|a_2| \leq \min\left\{\frac{2(1-\beta)}{1+\lambda}, \sqrt{\frac{2(1-\beta)}{1+2\lambda-\lambda^2}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{4(1-\beta)^2}{(1+\lambda)^2} + \frac{1-\beta}{1+2\lambda}, \frac{2(1-\beta)}{1+2\lambda-\lambda^2}\right\}.$$

Remark 2. *Corollary 6 and Corollary 7 provide an improvement of the estimates obtained by Keerthi and Raja [14].*

Taking $n = 0$ or $n = 0$ and $\lambda = 1$ in Theorem 2, we get

Corollary 8. *(see [23]) If* $f \in B_\Sigma(h, \lambda)$ *then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2 + |p'(0)|^2}{2(1+\lambda)^2}}, \sqrt{\frac{|h''(0)| + |p''(0)|}{4(1+2\lambda)}}\right\} \tag{29}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2 + |p'(0)|^2}{2(1+\lambda)^2} + \frac{|h''(0)| + |p''(0)|}{4(1+2\lambda)}, \frac{|h''(0)|}{2(1+2\lambda)}\right\}. \tag{30}$$

Corollary 9. *(see [23]) If* $f \in H_\Sigma(h)$ *then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2 + |p'(0)|^2}{8}}, \sqrt{\frac{|h''(0)| + |p''(0)|}{12}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2 + |p'(0)|^2}{8} + \frac{|h''(0)| + |p''(0)|}{12}, \frac{|h''(0)|}{6}\right\}.$$

Corollary 10. *If let*

$$h(z) = p(z) = \left(\frac{1+z}{1-z}\right)^\alpha = 1 + 2\alpha z + 2\alpha^2 z^2 + \ldots \qquad (0 < \alpha \leq 1),$$

then inequalities (17) and (18) become

$$|a_2| \leq \min\left\{\frac{2\alpha}{(1+\lambda)2^n}, \sqrt{\frac{2}{(1+2\lambda)3^n}}\alpha\right\}$$

and

$$|a_3| \leq \min\left\{\frac{4\alpha^2}{(1+\lambda)^2 2^{2n}} + \frac{2\alpha^2}{(1+2\lambda)3^n}, \frac{2\alpha^2}{(1+2\lambda)3^n}\right\}.$$

Corollary 11. *If let*

$$h(z) = p(z) = \frac{1 + (1-2\beta)z}{1-z} = 1 + 2(1-\beta)z + 2(1-\beta)z^2 + \cdots \qquad (0 \leq \beta < 1),$$

then inequalities (17) and (18) become

$$|a_2| \leq \min\left\{\frac{2(1-\beta)}{(1+\lambda)2^n}, \sqrt{\frac{2(1-\beta)}{(1+2\lambda)3^n}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{4(1-\beta)^2}{(1+\lambda)^2 2^{2n}} + \frac{2(1-\beta)}{(1+2\lambda)3^n}, \frac{2(1-\beta)}{(1+2\lambda)3^n}\right\}.$$

Remark 3. *Corollary 10 and Corollary 11 provide an improvement of the estimates obtained by Porwal and Darus [20].*

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