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LACUNARY STRONGLY ALMOST SUMMABLE SEQUENCE SPACES DEFINED BY IDEAL AND MODULUS FUNCTION

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Abstract. In the present article, we define a certain type of sequence spaces:- $w_\theta^f[\mathcal{I},p]_0$, $w_\theta^f[\mathcal{I},p]$ and $w_\theta^f[\mathcal{I},p]_\infty$.Which are defined by combining the concepts of modulus functions, lacunary sequence and $\mathcal{I}-$convergence. We also examined some topological properties of the resulting sequence spaces. In the last section, we introduce the concept of $\mathcal{I}-$lacunary almost statistical convergence and find out a condition under which this convergence coincides with $w_\theta^f[\mathcal{I},p]$.

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1. Introduction and Background

Let ℓ_∞ and C be the Banach spaces of real bounded and convergent sequences with the usual supremum norm. In Banach [1], a linear functional $\pounds$ on ℓ_∞ is said to be a Banach limit if it satisfies the following conditions:

(i) $\pounds(x) \geq 0$ when the sequence $x = (x_k)$ has $x_k \geq 0$ for all k, (ii) $\pounds(e) = 1$, where $e = (1, 1, 1, \cdots)$, (iii) $\pounds(Dx) = \pounds(x)$, where D is the shift operator defined by $(Dx_k = x_{k+1})$.

Let $\mathfrak{B}$ be the set of all Banach limits on ℓ_∞. A sequence x is said to be almost convergent to a number L if $\pounds(x) = L$ for all $\pounds \in \mathfrak{B}$. Lorentz[15] has shown that x is almost convergent to L if and only if

$$t_{k_m} = t_{k_m}(x) = \frac{x_m + x_{m+1} + \cdots + x_{m+k}}{k+1} \to L \text{ as } k \to \infty,$$

uniformly to m.

Maddox[16] and Freedman et.al.[9] introduced the concept of strongly almost convergence. Further, Das and Sahoo[3] defined the sequence space

$$[w(p)] = \{x \in W : \frac{1}{n+1}\sum_{k=0}^{n} |t_{k_m}(x-\ell)|^{p_k} \to 0 \text{ as } n \to \infty, \}$$

uniformly to m and investigated some of its properties.

The notion of statistical convergence has been introduced by Fast[8] in 1951 and has been developed extensively in different directions by Šalát[22], Fridy[10], Connor[2], Maddox[18] and many others.

A number sequence $x = (x_k)$ is said to be statistically convergent to a number L (denoted by $S - \lim_{k\to\infty} x_k = L$) provided that for every $\epsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leq n : |x_k - L| \geq \epsilon\}| = 0,$$

where the vertical bars denote the cardinality of the enclosed set. Let S denotes the set of all statistically convergent sequences of numbers.

By a lacunary sequence, we mean an increasing sequence $\theta = (k_r)$ of positive integers such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$, where the ratio $\frac{k_r}{k_{r-1}}$ is denoted by q_r.

Using lacunary sequence, Fridy and Orhan [11] generalized statistical convergence as follows:

Let $\theta = (k_r)$ be a lacunary sequence. A sequence $x = (x_k)$ of numbers is said to be lacunary statistically convergent to a number L (denoted by $S_\theta - \lim_{k\to\infty} x_k = L$) if for each $\epsilon > 0$,

$$\lim_{r\longrightarrow\infty} \frac{1}{h_r}|\{k \in I_r : |x_k - L| \geq \epsilon\}| = 0.$$

Let S_θ denotes the set of all lacunary statistically convergent sequences of numbers. Recently, lacunary sequence it has been studied by various authors (for instance [12], [20] and [7]).

A family of sets $\mathcal{I} \subset \mathcal{P}(\mathbb{N})$ is called an ideal in $\mathbb{N}$ if and only if (i) $\emptyset \in \mathcal{I}$;(ii) For each $A, B \in \mathcal{I}$ we have $A \cup B \in \mathcal{I}$; (iii) For $A \in \mathcal{I}$ and $B \subset A$ we have $B \in \mathcal{I}$.

A non-empty family of sets $\mathcal{F} \subset \mathcal{P}(\mathbb{N})$ is called a filter on $\mathbb{N}$ if and only if (i) $\emptyset \notin \mathcal{F}$;(ii) For each $A, B \in \mathcal{F}$ we have $A \cap B \in \mathcal{F}$;(iii) For $A \in \mathcal{F}$ and $B \supset A$ we have $B \in \mathcal{F}$.

An ideal $\mathcal{I}$ is called non-trivial if $\mathcal{I} \neq \emptyset$ and $\mathbb{N} \notin \mathcal{I}$.

It immediately implies that $\mathcal{I} \subset \mathcal{P}(\mathbb{N})$ is a non-trivial ideal if and only if the class $\mathcal{F} = \mathcal{F}(\mathcal{I}) = \{\mathbb{N} - A : A \in \mathcal{I}\}$ is a filter on $\mathbb{N}$. The filter $\mathcal{F} = \mathcal{F}(\mathcal{I})$ is called the filter associated with the ideal $\mathcal{I}$.

A non-trivial ideal $\mathcal{I} \subset \mathcal{P}(\mathbb{N})$ is called an admissible ideal in $\mathbb{N}$ if and only if it contains all singletons i.e. if it contains $\{\{n\} : n \in \mathbb{N}\}$. Throughout the paper, $\mathcal{I}$ is considered as a non-trivial admissible ideal.

Using the above terminology, Kostyrko et.al.[14] defined $\mathcal{I}-$convergence as follows:

A sequence $x = (x_k)$ in X is said to be $\mathcal{I}-$convergent to $\xi \in X$ if for each $\epsilon > 0$, the set $A(\epsilon) = \{k \in \mathbb{N} : |x_k - \xi| \geq \epsilon\} \in \mathcal{I}$. In this case, we write $\mathcal{I} - \lim_{k\to\infty} x_k = \xi$. The detailed history and development of this convergence can be found in ([5], [6], [13] and [4]).

The following inequality will be used throughout the paper. Let $p = (p_k)$ be a positive sequence of real numbers with $0 < p_k \leq \sup_k p_k = H$, $C = \max(1, 2^{H-1})$. Then for a_k, $b_k \in \mathbb{C}$, we have

$$|a_k + b_k|^{p_k} \leq C\{|a_k|^{p_k} + |b_k|^{p_k}\}, \tag{1}$$

for all $k \in \mathbb{N}$.

The notion of modulus function was introduced by Nakano[19] and now we recall that a modulus function f is a function from $[0, \infty)$ to $[0, \infty)$ such that

(i) $f(x) = 0$ if and only if $x = 0$, ii) $f(x + y) \leq f(x) + f(y)$ for all $x, y \geq 0$, iii) f is increasing, iv) f is continuous from right at 0.

It follows that f must be continuous everywhere on $[0, \infty)$. A modulus function may be bounded or unbounded. Subsequently, the notion of modulus function was used to define sequence spaces by Ruckle[21], Maddox[17], Pehilvan and Fisher[20], Savas[23], Et.and Gokhan[7] and many others. The following well-known lemma is required for establishing a very important result in our article.

Let f be a modulus function and let $0 < \delta < 1$. Then for each $x > \delta$ we have $f(x) \leq \frac{2.f(1)x}{\delta}$.

2. Main Results

In this section, we define a certain type of ideal convergent sequence spaces, where $w(X)$ denotes the space of all sequences $x = (x_k) \in X$.

Definition 1. *Let $\mathcal{I}$ be an admissible ideal, f be a modulus function and $p = (p_k)$ be any sequence of positive real numbers. For each $\epsilon > 0$, we define the following sequence spaces:*

$$w_\theta^f[\mathcal{I}, p]_0 = \left\{ x \in w(X) : \{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} [f(|t_{k_n}(x)|)]^{p_k} \geq \epsilon\} \in \mathcal{I}, \text{uniformly in n} \right\},$$

$$w_\theta^f[\mathcal{I}, p] = \left\{ x \in w(X) : \exists\ \ell > 0, \{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} [f(|t_{k_n}(x - \ell)|)]^{p_k} \geq \epsilon\} \in \mathcal{I} \right\},$$

and

$$w_\theta^f[\mathcal{I},p]_\infty = \left\{ x \in w(X) : \exists K > 0 \text{ such that } \{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k \in I_r}[f(|t_{k_n}(x)|)]^{p_k} \geq K\} \in \mathcal{I} \right\},$$

uniformly in n. *We can write it as* $x = (x_k) \in w_\theta^f[\mathcal{I},p]$ *or* $x_k \to L(w_\theta^f[\mathcal{I},p])$.

Remark 1. *If we take particularly* $f(x) = x$ *in the above definition, then we obtain* $w_\theta[\mathcal{I},p]_0$, $w_\theta[\mathcal{I},p]$ *instead of* $w_\theta^f[\mathcal{I},p]_0$ *and* $w_\theta^f[\mathcal{I},p]$ *respectively.*

Remark 2. *When we choose* $p_k = 1$ *for all* $k \in \mathbb{N}$, *then the spaces* $w_\theta^f[\mathcal{I},p]_0$, $w_\theta^f[\mathcal{I},p]$ *and* $w_\theta^f[\mathcal{I},p]_\infty$ *reduce to the spaces* $w_\theta^f[\mathcal{I}]_0$, $w_\theta^f[\mathcal{I}]$ *and* $w_\theta^f[\mathcal{I}]_\infty$.

Theorem 1. *If* $p = (p_k)$ *be a bounded sequence and* f *be a modulus function, then* $w_\theta^f[\mathcal{I},p]_0$, $w_\theta^f[\mathcal{I},p]$ *and* $w_\theta^f[\mathcal{I},p]_\infty$ *are linear spaces over* $\mathbb{C}$.

Proof. We have

$$|a_k + b_k|^{p_k} \leq C\{|a_k|^{p_k} + |b_k|^{p_k}\}, \tag{2}$$

where $\sup_k p_k = H$ and $C = \max(1, 2^{H-1})$. We shall prove the assertion for $w_\theta^f[\mathcal{I},p]_0$, the others can be treated similarly. Let $x = (x_k)$, $y = (y_k) \in w_\theta^f[\mathcal{I},p]_0$. Then for every $\epsilon > 0$, the sets

$$A_\theta(\epsilon) = \{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k \in I_r}[f(|t_{k_n}(x)|)]^{p_k} \geq \frac{\epsilon}{2}\}, \tag{3}$$

$$B_\theta(\epsilon) = \{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k \in I_r}[f(|t_{k_n}(y)|)]^{p_k} \geq \frac{\epsilon}{2}\} \tag{4}$$

belong to $\mathcal{I}$, uniformly in n.

Let α, $\beta \in \mathbb{C}$, then we have

$$\frac{1}{h_r}\sum_{k \in I_r}[f(|t_{k_n}(\alpha.x + \beta.y)|)]^{p_k} \leq \frac{1}{h_r}\sum_{k \in I_r}[f(|\alpha|.|t_{k_n}(x)|) + f(|\beta|.|t_{k_n}(y)|)]^{p_k}$$

$$\leq C.(K_\alpha)^H.\frac{1}{h_r}\sum_{k \in I_r}[f(|t_{k_n}(x)|)]^{p_k} + C.(K_\beta)^H.\frac{1}{h_r}\sum_{k \in I_r}[f(|t_{k_n}(y)|)]^{p_k} \quad \text{by (2)},$$

where $|\alpha| \leq K_\alpha$ and $|\beta| \leq K_\beta$.

Then for given $\epsilon > 0$, we have the following inclusion relations

$$\begin{aligned} &\{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} [f(|t_{k_n}(\alpha.x + \beta.y)|)]^{p_k} \geq \epsilon\} \\ \subseteq &\{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} [f(|t_{k_n}(x)|)]^{p_k} \geq \frac{\epsilon}{2C.(K_\alpha)^H}\} \\ \cup &\{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} [f(|t_{k_n}(y)|)]^{p_k} \geq \frac{\epsilon}{2C.(K_\beta)^H}\} \end{aligned}$$

uniformly in n.

By using (3) and (4), the set $\{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} [f(|t_{k_n}(\alpha.x + \beta.y)|)]^{p_k} \geq \epsilon\} \in \mathcal{I}$. This completes the proof.

Theorem 2. *Let f' and f'' are modulus functions. If*

$$\limsup_{t \to \infty} \frac{f'(t)}{f''(t)} = M > 0,$$

then $w_\theta^{f'}[\mathcal{I}, p] \subset w_\theta^{f''}[\mathcal{I}, p]$.

Proof. We assume that $\limsup_{t \to \infty} \frac{f'(t)}{f''(t)} = M$, then there exists an integer $K > 0$ such that $f'(t) \geq K.f''(t)$ for all $t \geq 0$.
Which implies that

$$\frac{1}{h_r} \sum_{k \in I_r} [f'(|t_{k_n}(x - \ell)|)]^{p_k} \geq (K)^H . \frac{1}{h_r} \sum_{k \in I_r} [f''(|t_{k_n}(x - \ell)|)]^{p_k}.$$

Then for any $\epsilon > 0$, we have

$$\{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} [f''(|t_{k_n}(x - \ell)|)]^{p_k} \geq \epsilon\} \subseteq \{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} [f'(|t_{k_n}(x - \ell)|)]^{p_k} \geq \epsilon.(K)^H\},$$

uniformly in n.
Since $x \in w_\theta^{f'}[\mathcal{I}, p]$, therefore by above containment it follows that $x \in w_\theta^{f''}[\mathcal{I}, p]$.

Theorem 3. *If f, f' and f'' are modulus functions, then*
(i) $w_\theta^{f'}[\mathcal{I}, p] \subset w_\theta^{f \circ f'}[\mathcal{I}, p]$,
(ii) $w_\theta^{f'}[\mathcal{I}, p] \cap w_\theta^{f''}[\mathcal{I}, p] \subset w_\theta^{f' + f''}[\mathcal{I}, p]$.

Proof. (i) We suppose that $x = (x_k) \in w_\theta^{f'}[\mathcal{I}, p]$. Let $\epsilon > 0$, we choose $\delta \in (0, 1)$ such that $f(t) < \epsilon$ for all $0 < t < \delta$. Since $x \in w_\theta^{f'}[\mathcal{I}, p]$ such that

$$A_\theta(\delta) = \{r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} [f'(|t_{k_n}(x - \ell)|)]^{p_k} \geq \delta\} \in \mathcal{I}, \tag{5}$$

uniformly in n.
On the other hand, we have

$$\begin{aligned}\frac{1}{h_r} \sum_{k \in I_r} [f \circ f'(|t_{k_n}(x - \ell)|)]^{p_k} &= \frac{1}{h_r} \sum_{k \in I_r \& [f'(|t_{k_n}(x-\ell)|)]^{p_k} < \delta} [f \circ f'(|t_{k_n}(x - \ell)|)]^{p_k} \\ &+ \frac{1}{h_r} \sum_{k \in I_r \& [f'(|t_{k_n}(x-\ell)|)]^{p_k} \geq \delta} [f \circ f'(|t_{k_n}(x - \ell)|)]^{p_k} \\ &\leq (\epsilon)^H + \max(1, (2.\frac{f(1)}{\delta})^H).\frac{1}{h_r} \sum_{k \in I_r} [f'(|t_{k_n}(x - \ell)|)]^{p_k}.\end{aligned}$$

By using (5), we have $x \in w_\theta^{f \circ f'}[\mathcal{I}, p]$.
(ii) The result of the theorem can be proved by using the following inequality

$$\begin{aligned}\frac{1}{h_r} \sum_{k \in I_r} [(f' + f'')(|t_{k_n}(x - \ell)|)]^{p_k} &\leq \frac{C}{h_r} \sum_{k \in I_r} [f'(|t_{k_n}(x - \ell)|)]^{p_k} \\ &+ \frac{C}{h_r} \sum_{k \in I_r} [f''(|t_{k_n}(x - \ell)|)]^{p_k},\end{aligned}$$

where $\sup_k p_k = H$ and $C = \max(1, 2^{H-1})$.

Theorem 4. *Let f be a modulus function. If $p = (p_k)$ be a sequence of positive real numbers, then $w_\theta[\mathcal{I}, p] \subseteq w_\theta^f[\mathcal{I}, p]$.*

Proof. This result can be proved by using the techniques similar to those used in theorem 3(i).

Theorem 5. *Let f be a modulus function. If $\limsup_{t \to \infty} \frac{f(t)}{t} = L > 0$, then*

$$w_\theta^f[\mathcal{I}, p] \subseteq w_\theta[\mathcal{I}, p].$$

Proof. We have $\limsup_{t \to \infty} \frac{f(t)}{t} = L > 0$, then there exists a constant $K > 0$ such that $f(t) \geq K.t$ for all $t \geq 0$.
Which implies that

$$\frac{1}{h_r} \sum_{k \in I_r} [f(|t_{k_n}(x - \ell)|)]^{p_k} \geq (K)^H.\frac{1}{h_r} \sum_{k \in I_r} [|t_{k_n}(x - \ell)|]^{p_k}.$$

This gives the result.

Theorem 6. *If $0 < p_k \leq q_k$ and $(\frac{q_k}{p_k})$ be bounded, then $w_\theta^f[\mathcal{I}, q] \subset w_\theta^f[\mathcal{I}, p]$.*

Proof. The proof of this theorem is omitted.

3. $[S_\theta(\mathcal{I})]-$convergence

In this section, we introduce the concept of $\mathcal{I}-$lacunary almost statistical convergence and also obtain a condition under which the class $[S_\theta(\mathcal{I})]$ of all $\mathcal{I}-$lacunary almost statistically convergent sequences coincides with the sequence space $w_\theta^f[\mathcal{I}, p]$.

Definition 2. *A sequence $x = (x_k)$ is said to be $\mathcal{I}-$lacunary almost statistically convergent or $[S_\theta(\mathcal{I})]-$convergent to a number ℓ provided that for every $\epsilon > 0$ and $\delta > 0$*

$$\left\{r \in \mathbb{N} : \frac{1}{h_r}|\{k \in I_r : |t_{k_n}(x - \ell)| \geq \epsilon\}| \geq \delta\right\} \in \mathcal{I}, \textit{uniformly in } n.$$

In this case, we write $x_k \to \ell([S_\theta(\mathcal{I})])$ or $[S_\theta(\mathcal{I})] - \lim_{k\to\infty} x_k = \ell$. The set of all $\mathcal{I}-$lacunary almost statistically convergent sequences is denoted by $[S_\theta(\mathcal{I})]$.

Theorem 7. *Let f be a modulus function and $p = (p_k)$ be a sequence of positive real numbers. If $0 < inf_k p_k = h \leq p_k \leq sup_k p_k = H < \infty$, then $w_\theta^f[\mathcal{I}, p] \subset [S_\theta(\mathcal{I})]$.*

Proof. Suppose $x \in w_\theta^f[\mathcal{I}, p]$ and $\epsilon > 0$ be given. Then we have

$$\frac{1}{h_r}\sum_{k\in I_r}[f(|t_{k_n}(x - \ell)|)]^{p_k} = \frac{1}{h_r}\sum_{k\in I_r \& |t_{k_n}(x-\ell)|\geq\epsilon}[f(|t_{k_n}(x - \ell)|)]^{p_k}$$
$$+\frac{1}{h_r}\sum_{k\in I_r \& |t_{k_n}(x-\ell)|<\epsilon}[f(|t_{k_n}(x - \ell)|)]^{p_k}$$

$$\geq \frac{1}{h_r}\sum_{k\in I_r \& |t_{k_n}(x-\ell)|\geq\epsilon}[f(|t_{k_n}(x - \ell)|)]^{p_k} \geq \frac{1}{h_r}\sum_{k\in I_r}[f(\epsilon)]^{p_k}$$

$$\geq \frac{1}{h_r}\sum_{k\in I_r}\min([f(\epsilon)]^h, [f(\epsilon)]^H) \geq \frac{1}{h_r}|\{k \in I_r : |t_{k_n}(x - \ell)| \geq \epsilon\}|.\min([f(\epsilon)]^h, [f(\epsilon)]^H).$$

Then for $\delta > 0$ and uniformly in n, we have

$$\left\{r \in \mathbb{N} : \frac{1}{h_r}|\{k \in I_r : |t_{k_n}(x - \ell)| \geq \epsilon\}| \geq \delta\right\} \subseteq \{r \in \mathbb{N} : \frac{1}{h_r}\sum_{k\in I_r}[f(|t_{k_n}(x - \ell)|)]^{p_k} \geq K.\delta\},$$

where $K = \min([f(\epsilon)]^h, [f(\epsilon)]^H)$.
Since $x_k \to \ell(w_\theta^f[\mathcal{I}, p])$ so that $[S_\theta(\mathcal{I})] - \lim_{k\to\infty} x_k = \ell$.

Theorem 8. *Let f be a bounded modulus function and $0 < inf_k p_k = h \leq p_k \leq sup_k p_k = H < \infty$, then $[S_\theta(\mathcal{I})] \subset w_\theta^f[\mathcal{I}, p]$.*

Proof. Using the same technique of theorem 3.3 of [7], it is easy to prove this theorem.

Theorem 9. *If $0 < inf_k p_k = h \leq p_k \leq sup_k p_k = H < \infty$, then $[S_\theta(\mathcal{I})] = w_\theta^f[\mathcal{I}, p]$ if and only if f is bounded.*

Proof. This part is the direct consequence of theorem 7 and 8.
Conversely : Suppose f is unbounded defined by $f(k) = k$ for all $k \in \mathbb{N}$ and $\theta = (2^r)$ be a lacunary sequence. We take a fixed $A \in \mathcal{I}$ and define $x = (x_k)$ as follows:

$$x_k = \begin{cases} k, & \text{for } r \notin A, \ 2^{r-1} + 1 \leq k \leq 2^{r-1} + [\sqrt{h_r}], \\ k, & \text{for } r \in A, \ 2^{r-1} < k \leq 2^{r-1} + h_r, \\ 0, & \text{otherwise,} \end{cases}$$

where $I_r = (2^{r-1}, 2^r]$ and $h_r = 2^r - 2^{r-1}$.
Then for given $\epsilon > 0$, we have

$$\lim_{r\to\infty} \frac{1}{h_r} |\{k \in I_r : |t_{k_n}(x - 0)| \geq \epsilon\}| \leq \frac{[\sqrt{h_r}]}{h_r} \to 0$$

for $r \notin A$ and uniformly in n.
Hence for $\delta > 0$, there exists a positive integer r_0 such that

$$\frac{1}{h_r} |\{k \in I_r : |t_{k_n}(x - 0)| \geq \epsilon\}| < \delta \text{ for } r \notin A \text{ and } r \geq r_0.$$

Now, we have

$$\{r \in \mathbb{N} : \frac{1}{h_r} |\{k \in I_r : |t_{k_n}(x - 0)| \geq \epsilon\}| \geq \delta\} \subset \{A \cup (1, 2, \cdots r_0 - 1)\},$$

uniformly in n.
Since $\mathcal{I}$ be an admissible ideal, It follows that $[S_\theta(\mathcal{I})] - \lim_{k\to\infty} x_k = 0$.

If we take $p_k = 1$ for all $k = 1, 2, \cdots$ and $t_{0_n}(x) = x_n$, then $x = (x_k) \notin w_\theta^f[\mathcal{I}, p]$. This contradicts the fact $[S_\theta(\mathcal{I})] = w_\theta^f[\mathcal{I}, p]$.

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