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NAN-HUR-STABILITY OF AN ADDITIVE MAPPING

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Abstract. In this paper, using the fixed point alternative approach and direct method we investigate the generalized Hyers-Ulam stability of the following additive functional equation

$$H\left(\frac{x+y}{2}+z\right)=\frac{H(x)+H(y)}{2}+H(z) \tag{1}$$

in non-Archimedean normed spaces.
The concept of Hyers-Ulam-Rassias stability (briefly, HUR-stability) originated from Th. M. Rassias stability theorem that appeared in his paper: On the stability of the linear mapping in Banach spaces, Proc. Amer. Math. Soc. 72 (1978), 297-300.

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1. Introduction and preliminaries

A *valuation* is a function $|\cdot|$ from a field $\mathbb{K}$ into $[0,\infty)$ such that 0 is the unique element having the 0 valuation, $|rs|=|r||s|$ and the triangle inequality holds, i.e.,

$$|r+s|\leq \max\{|r|,|s|\}.$$

A field $\mathbb{K}$ is called a *valued field* if $\mathbb{K}$ carries a valuation. The usual absolute values of $\mathbb{R}$ and $\mathbb{C}$ are examples of valuations.
Let us consider a valuation which satisfies a stronger condition than the triangle inequality. If the triangle inequality is replaced by $|r+s|\leq \max\{|r|,|s|\}$, for all $r,s\in\mathbb{K}$, then the function $|\cdot|$ is called a *non-Archimedean valuation* and the field is called a *non-Archimedean field*. Clearly, $|1|=|-1|=1$ and $|n|\leq 1$ for all $n\geq 1$. A trivial example of a non-Archimedean valuation is the function $|\cdot|$ taking everything except for 0 into 1 and $|0|=0$.

Definition 1. *Let X be a vector space over a field $\mathbb{K}$ with a non-Archimedean valuation $|\cdot|$. A function $\|\cdot\| : X \to [0,\infty)$ is called a non-Archimedean norm if the following conditions hold:*
(a) $\|x\| = 0$ *if and only if* $x = 0$ *for all* $x \in X$*;*
(b) $\|rx\| = |r|\|x\|$ *for all* $r \in \mathbb{K}$ *and* $x \in X$*;*
(c) *the strong triangle inequality holds:*

$$\|x + y\| \leq max\{\|x\|, \|y\|\}$$

for all $x, y \in X$*.*
Then $(X, \|\cdot\|)$ *is called a non-Archimedean normed space(briefly NAN-spaces).*

Definition 2. *Let* $\{x_n\}$ *be a sequence in a non-Archimedean normed space* X*.*
(a) *A sequence* $\{x_n\}_{n=1}^{\infty}$ *in a non-Archimedean space is a Cauchy sequence iff, the sequence* $\{x_{n+1} - x_n\}_{n=1}^{\infty}$ *converges to zero.*
(b) *The sequence* $\{x_n\}$ *is said to be convergent if, for any* $\varepsilon > 0$*, there are a positive integer* N *and* $x \in X$ *such that* $\|x_n - x\| \leq \varepsilon$*, for all* $n \geq N$*. Then the point* $x \in X$ *is called the limit of the sequence* $\{x_n\}$*, which is denote by* $\lim_{n\to\infty} x_n = x$*.*
(c) *If every Cauchy sequence in* X *converges, then the non-Archimedean normed space* X *is called a non-Archimedean Banach space.*

Definition 3. *Let* X *be a set. A function* $d : X \times X \to [0,\infty]$ *is called a generalized metric on* X *if* d *satisfies the following conditions:*
(a) $d(x, y) = 0$ *if and only if* $x = y$ *for all* $x, y \in X$*;*
(b) $d(x, y) = d(y, x)$ *for all* $x, y \in X$*;*
(c) $d(x, z) \leq d(x, y) + d(y, z)$ *for all* $x, y, z \in X$*.*

Theorem 1. *Let (X,d) be a complete generalized metric space and* $J : X \to X$ *be a strictly contractive mapping with Lipschitz constant* $L < 1$*. Then, for all* $x \in X$*, either*

$$d(J^n x, J^{n+1} x) = \infty \tag{2}$$

for all nonnegative integers n *or there exists a positive integer* n_0 *such that*
(a) $d(J^n x, J^{n+1} x) < \infty$ *for all* $n_0 \geq n_0$*;*
(b) *the sequence* $\{J^n x\}$ *converges to a fixed point* y^* *of* J*;*
(c) y^* *is the unique fixed point of* J *in the set* $Y = \{y \in X : d(J^{n_0} x, y) < \infty\}$*;*
(d) $d(y, y^*) \leq \frac{1}{1-L} d(y, Jy)$ *for all* $y \in Y$*.*

The first stability problem concerning group homomorphisms was raised by Ulam [26] in 1940. In the next year, Hyres [14] gave a positive answer to the above question for additive groups under the assumption that the groups are Banach spaces. In 1978, Rassias [20] proved a generalization of Hyres' theorem for additive mappings.

Theorem 2. *(Th.M. Rassias) Let $f : X \to Y$ be a mapping from a normed vector space X into a Banach space Y subject to the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^r + \|y\|^r) \tag{3}$$

for all $x, y \in X$, where ϵ and r are constants with $\epsilon > 0$ and $r < 1$. Then the limit $L(x) := \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$ exists for all $x \in X$ and $L : X \to Y$ is the unique linear mapping which satisfies

$$\|f(x) - L(x)\| \leq \frac{2\epsilon}{2 - 2^p}\|x\|^r \tag{4}$$

for all $x \in X$. Also, if for each $x \in X$ the function $f(tx)$ is continuous in $t \in \mathbb{R}$, then L is $\mathbb{R}$-linear.

The result of Rassias has influenced the development of what is now called the *Hyers-Ulam-Rassias stability problem* for functional equations. In 1994, a generalization of Rassias's theorem was obtained by Găvruta [12] by replacing the bound $\epsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x, y)$.
The functional equation $f(x+y) + f(x-y) = 2f(x) + 2f(y)$ is called a *quadratic functional equation*. In particular, every solution of the quadratic functional equation is said to be a *quadratic mapping*. In 1983, a generalized Hyers-Ulam stability problem for the quadratic functional equation was proved by Skof [25] for mappings $f : X \to Y$, where X is a normed space and Y is a Banach space. In 1984, Cholewa [5] noticed that the theorem of Skof is still true if the relevant domain X is replaced by an Abelian group and, in 2002, Czerwik [6] proved the generalized Hyers-Ulam stability of the quadratic functional equation.
The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem ([1]- [4],[7]–[11], [17], [19]- [24]).
In 1897, Hensel [13] has introduced a normed space which does not have the Archimedean property. It turned out that non-Archimedean spaces have many nice applications ([15], [16], [18]).
In this paper, we prove the generalized Hyers-Ulam stability of the following functional equation

$$H\Big(\frac{x+y}{2} + z\Big) = \frac{H(x) + H(y)}{2} + H(z) \tag{5}$$

in non-Archimedean spaces.

2. Non-Archimedean stability of Eq.(5): a fixed point method

Throughout this section, using the fixed point alternative approach we prove the generalized Hyers-Ulam stability of functional equation (5) in non-Archimedean normed spaces.
Also, in this section we assume that X is a non-Archimedean normed space and that Y is a complete non-Archimedean space. Let $|2| \neq 1$.

Definition 4. *Let $\pounds : X^3 \to [0, \infty)$ be a function such that there exists an $L < 1$ with*

$$|2|\pounds\Big(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\Big) \leq L\pounds(x, y, z) \tag{6}$$

for all $x, y, z \in X$. Let $H : X \to Y$ be a mapping satisfying

$$\Big\|H\Big(\frac{x+y}{2} + z\Big) - \frac{H(x) + H(y)}{2} - H(z)\Big\|_Y \leq \pounds(x, y, z) \tag{7}$$

for all $x, y, z \in X$. Then there is a unique additive mapping $R : X \to Y$ such that

$$\|H(x) - K(x)\|_Y \leq \frac{L\pounds(x, x, x)}{|2|(1 - L)} \tag{8}$$

Proof. Putting $x = y = z$ in (7), we have

$$\Big\|2H\Big(\frac{x}{2}\Big) - H(x)\Big\|_Y \leq \pounds\Big(\frac{x}{2}, \frac{x}{2}, \frac{x}{2}\Big) \tag{9}$$

for all $x \in X$. Consider the set $S := \{G : X \to Y\}$ and the generalized metric d in S defined by

$$d(H, G) = \inf\Big\{\mu \in (0, +\infty) : \|G(x) - H(x)\|_Y \leq \mu\pounds(x, x, x), \forall x \in X\Big\}, \tag{10}$$

where $\inf \emptyset = +\infty$. It is easy to show that (S, d) is complete (see [17], Lemma 2.1). Now, we consider a linear mapping $J : S \to S$ such that

$$JG(x) := 2G\Big(\frac{x}{2}\Big) \tag{11}$$

for all $x \in X$. Let $G, K \in S$ be such that $d(G, H) = \lambda$. Then $\|G(x) - H(x)\|_Y \leq \epsilon\pounds(x, x, x)$ for all $x \in X$ and so

$$\|JG(x) - JH(x)\|_Y = \Big\|2G\Big(\frac{x}{2}\Big) - 2H\Big(\frac{x}{2}\Big)\Big\|_Y \leq |2|\lambda\pounds\Big(\frac{x}{2}, \frac{x}{2}, \frac{x}{2}\Big) \leq \lambda L$$

for all $x \in X$. Thus $d(G, H) = \epsilon$ implies that $d(JG, JH) \leq L\lambda$. This means that

$$d(JG, JH) \leq Ld(G, H) \tag{12}$$

for all $G, H \in S$. It follows from (9) that

$$d(H, JH) \leq \frac{L}{|2|} < \infty. \tag{13}$$

By Theorem (1), there exists a mapping $K : X \to Y$ satisfying the following:
(1) K is a fixed point of J, that is,

$$K\Big(\frac{x}{2}\Big) = \frac{K(x)}{2} \tag{14}$$

for all $x \in X$. The mapping K is a unique fixed point of J in the set

$$\Omega = \{H \in S : d(G, H) < \infty\}. \tag{15}$$

This implies that K is a unique mapping satisfying (14) such that there exists $\mu \in (0, \infty)$ satisfying

$$\|H(x) - K(x)\|_Y \leq \mu \pounds(x, x, x) \tag{16}$$

for all $x \in X$.
(2) $d(J^n H, K) \to 0$ as $n \to \infty$. This implies the equality

$$\lim_{n\to\infty} 2^n H\Big(\frac{x}{2^n}\Big) = K(x) \tag{17}$$

for all $x \in X$.
(3) $d(H, K) \leq \frac{d(H,JH)}{1-L}$ with $H \in \Omega$, which implies the inequality

$$d(H, K) \leq \frac{L}{|2|(1 - L)}. \tag{18}$$

This implies that the inequality (8) holds. On the other hand, By (7), we get

$$\Big\|K\Big(\frac{x+y}{2} + z\Big) - \frac{K(x) + K(y)}{2} - K(z)\Big\|_Y \leq \lim_{n\to\infty} L^n \pounds(x, y, z)$$

for all $x, y, z \in X$ and $n \in \mathbb{N}$. Thus, the mapping $R : X \to Y$ is additive, as desired.

Corollary 3. *Let $\theta \geq 0$ and q be a real number with $0 < q < 1$. Let $H : X \to Y$ be a mapping satisfying*

$$\Big\|H\Big(\frac{x+y}{2} + z\Big) - \frac{H(x) + H(y)}{2} - H(z)\Big\|_Y \leq \theta(\|x\|^q + \|y\|^q + \|z\|^q) \tag{19}$$

for all $x, y, z \in X$. *Then*

$$K(x) = \lim_{n\to\infty} 2^n H\Big(\frac{x}{2^n}\Big) \tag{20}$$

exists for all $x \in X$ *and* $K : X \to Y$ *is a unique additive mapping such that*

$$\|H(x) - K(x)\| \le \frac{3\theta\|x\|^q}{|2|(|2|^{q-1} - 1)} \tag{21}$$

for all $x \in X$.

Proof. The proof follows from Theorem (4) by taking

$$\zeta(x, y, z) = \theta(\|x\|^q + \|y\|^q + \|z\|^q) \tag{22}$$

for all $x, y, z \in X$. In fact, if we choose $L = |2|^{1-q}$, then we get the desired result.

Similarly, we have the following and then we omit the proof.

Theorem 4. *Let* $\pounds : X^3 \to [0, \infty)$ *be a function such that there exists an* $L < 1$ *with*

$$\zeta(x, y, z) \le |2| L \pounds\Big(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\Big) \tag{23}$$

for all $x, y, z \in X$. *Let* $H : X \to Y$ *be a mapping satisfying*

$$\Big\|H\Big(\frac{x+y}{2} + z\Big) - \frac{H(x) + H(y)}{2} - H(z)\Big\|_Y \le \pounds(x, y, z) \tag{24}$$

for all $x, y, z \in X$. *Then*

$$K(x) = \lim_{n\to\infty} \frac{H(2^n x)}{2^n} \tag{25}$$

exists for all $x \in X$ *and defines a unique additive mapping* $K : X \to Y$ *such that*

$$\|H(x) - K(x)\| \le \frac{1}{|2| - |2|L} \pounds(x, x, x) \tag{26}$$

Proof. It follows from (9) that

$$\Big\|H(x) - \frac{1}{2}H(2x)\Big\| \le \frac{1}{|2|} \pounds(x, x, x) \tag{27}$$

for all $x \in X$. The rest of the proof is similar to the proof of Theorem (4).

Corollary 5. *Let $\theta \geq 0$ and $r_1, r_2, r_3 \in \mathbb{R}^+$ be real numbers with $r_1 + r_2 + r_3 > 1$. Let $H : X \to Y$ be a mapping satisfying*

$$\left\| H\Big(\frac{x+y}{2} + z\Big) - \frac{H(x) + H(y)}{2} - H(z) \right\|_Y \leq \theta(\|x\|^{r_1}.\|y\|^{r_2}.\|z\|^{r_3}) \tag{28}$$

for all $x, y, z \in X$. Then

$$K(x) = \lim_{n\to\infty} \frac{H(2^n x)}{2^n} \tag{29}$$

exists for all $x \in X$ and $K : X \to Y$ is a unique additive mapping such that

$$\|H(x) - K(x)\| \leq \frac{\theta \|x\|^{r_1+r_2+r_3}}{|2| - |2|^{r_1+r_2+r_3}} \tag{30}$$

for all $x \in X$.

Proof. The proof follows from Theorem (4) by taking

$$\pounds(x, y, z) = \theta(\|x\|^{r_1}.\|y\|^{r_2}.\|z\|^{r_3}) \tag{31}$$

for all $x, y, z \in X$. In fact, if we choose $L = |2|^{r_1+r_2+r_3-1}$, then we get the desired result.

3. Non-Archimedean Hyers-Ulam stability of Eq.(5): a direct method

Throughout this section, using direct method we prove the generalized Hyers-Ulam stability of functional equation (5) in non-Archimedean normed spaces.
Also, in this section we assume that G is a commutative semigroup and X is a complete non-Archimedean space.

Theorem 6. *Let $\omega : G^3 \to [0, +\infty)$ be a function such that*

$$\lim_{n\to\infty} |2|^n \omega\Big(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}\Big) = 0 \tag{32}$$

for all $x, y, z \in G$ and let for each $x \in G$ the limit

$$\nabla(x) = \lim_{n\to\infty} max\Big\{|2|^k \omega\Big(\frac{x}{2^k}, \frac{x}{2^k}, \frac{x}{2^k}\Big);\ 0 \leq k < n\Big\} \tag{33}$$

exists. Suppose that $Q : G \to X$ is a mapping satisfies

$$\left\| H\Big(\frac{x+y}{2} + z\Big) - \frac{H(x) + H(y)}{2} - H(z) \right\|_Y \leq \omega(x, y, z). \tag{34}$$

Then

$$\Re(x) := \lim_{n\to\infty} 2^n H\Big(\frac{x}{2^n}\Big) \tag{35}$$

exists for all $x \in G$ and defines an additive mapping $\Re : G \to X$ such that

$$||H(x) - \Re(x)|| \leq \nabla(x) \tag{36}$$

Moreover, if

$$\lim_{j\to\infty}\lim_{n\to\infty} max\Big\{|2|^k\omega\Big(\frac{x}{2^k}, \frac{x}{2^k}, \frac{x}{2^k}\Big);\ j \leq k < n+j\Big\} = 0 \tag{37}$$

then $\Re$ is the unique additive mapping satisfying (36).

Proof. Putting $x = y = z$ in (34), we get

$$\Big\|2H\Big(\frac{x}{2}\Big) - H(x)\Big\| \leq \omega\Big(\frac{x}{2}, \frac{x}{2}, \frac{x}{2}\Big) \tag{38}$$

for all $x \in G$. Replacing x by $\frac{x}{2^n}$ in (38), we obtain

$$\Big\|2^{n+1}H\Big(\frac{x}{2^{n+1}}\Big) - 2^n H\Big(\frac{x}{2^n}\Big)\Big\| \leq |2|^n\omega\Big(\frac{x}{2^n}, \frac{x}{2^n}, \frac{x}{2^n}\Big) \tag{39}$$

It follows from (32) and (39) that the sequence $\Big\{2^nH(\frac{x}{2^n})\Big\}_{n\geq 1}$ is a Cauchy sequence. Since X is complete, so $\Big\{2^nH(\frac{x}{2^n})\Big\}_{n\geq 1}$ is convergent. Set $\Re(x) := \lim_{n\to\infty} 2^n H\Big(\frac{x}{2^n}\Big)$. Using induction one can show that

$$\Big\|2^nH\Big(\frac{x}{2^n}\Big) - H(x)\Big\|_Y \leq max\Big\{|2|^k\omega\Big(\frac{x}{2^k}, \frac{x}{2^k}, \frac{x}{2^k}\Big);\ 0 \leq k < n\Big\}. \tag{40}$$

for all $n \in \mathbb{N}$ and all $x \in G$. By taking n to approach infinity in (40), and using (33), one obtains (36). By (32) and (42), we get

$$\Big\|K\Big(\frac{x+y}{2} + z\Big) - \frac{K(x)+K(y)}{2} - H(z)\Big\|_Y \leq \lim_{n\to\infty} |2|^n\omega\Big(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}\Big) = 0$$

for all $x, y, z \in G$. Therefore the function $\Re : G \to X$ satisfies (5). To prove the uniqueness property of $\Re$, Let $\Im : G \to X$ be another function satisfying (36). Then

$$\begin{aligned}\Big\|\Im(x) - \Re(x)\Big\|_Y &= \lim_{n\to\infty} |2|^n \Big\|\Im\Big(\frac{x}{2^n}\Big) - \Re\Big(\frac{x}{2^n}\Big)\Big\|_Y \\ &\leq \lim_{k\to\infty} |2|^n max\Big\{\Big\|\Im\Big(\frac{x}{2^n}\Big) - H\Big(\frac{x}{2^n}\Big)\Big\|_Y, \Big\|H\Big(\frac{x}{2^n}\Big) - \Re\Big(\frac{x}{2^n}\Big)\Big\|_Y\Big\} \\ &\leq \lim_{j\to\infty}\lim_{n\to\infty} max\Big\{|2|^k\omega\Big(\frac{x}{2^k}, \frac{x}{2^k}, \frac{x}{2^k}\Big);\ j \leq k < n+j\Big\} \\ &= 0\end{aligned}$$

for all $x \in G$. Therefore $\Im = \Re$, and the proof is complete.

Corollary 7. *Let $\xi : [0, \infty) \to [0, \infty)$ be a function satisfying*

$$\xi(|2|^{-1}t) \leq \xi(|2|^{-1})\xi(t) \quad (t \geq 0) \quad \xi(|2|^{-1}) < |2|^{-1} \tag{41}$$

Let $\kappa > 0$ and $H : G \to X$ be a mapping satisfying

$$\left\| H\Big(\frac{x+y}{2} + z\Big) - \frac{H(x) + H(y)}{2} - H(z) \right\|_Y \leq \kappa(\xi(|x|) + \xi(|y|) + \xi(|z|)). \tag{42}$$

for all $x, y, z \in G$. Then there exists a unique additive mapping $\Re : G \to X$ such that

$$||H(x) - \Re(x)|| \leq 3\kappa\xi(|x|) \tag{43}$$

Proof. Defining $\omega : G^3 \to [0, \infty)$ by $\omega(x, y, z) := \kappa(\xi(|x|) + \xi(|y|) + \xi(|z|))$. Applying Theorem (6), we get desired results.

Theorem 8. *Let $\omega : G^3 \to [0, +\infty)$ be a function such that*

$$\lim_{n\to\infty} |2|^{-n}\omega(2^n x, 2^n y, 2^n z) = 0 \tag{44}$$

for all $x, y, z \in G$ and let for each $x \in G$ the limit

$$\nabla(x) = \lim_{n\to\infty} max\Big\{|2|^{-k}\omega(2^k x, 2^k y, 2^k z);\ 0 \leq k < n\Big\} \tag{45}$$

exists. Suppose that $f : G \to X$ be a mapping satisfies

$$\left\| H\Big(\frac{x+y}{2} + z\Big) - \frac{H(x) + H(y)}{2} - H(z) \right\|_Y \leq \omega(x, y, z). \tag{46}$$

Then

$$\Re(x) := \lim_{n\to\infty} \frac{H(2^n x)}{2^n} \tag{47}$$

exists for all $x \in G$ and defines an additive mapping $\Re : G \to X$, such that

$$||H(x) - \Re(x)|| \leq |2|^{-1}\nabla(x) \tag{48}$$

Moreover, if

$$\lim_{j\to\infty} \lim_{n\to\infty} max\Big\{|2|^{-k}\omega(2^k x, 2^k x, 2^k x);\ j \leq k < n + j\Big\} = 0 \tag{49}$$

then $\Re$ is the unique mapping satisfying (48).

Proof. Putting $x = y = z$ in (46), we get

$$\left\| H(x) - \frac{H(2x)}{2} \right\| \leq |2|^{-1}\omega(x, x, x) \tag{50}$$

for all $x \in G$. Replacing x by $2^n x$ in (50), we obtain

$$\left\| \frac{H(2^n x)}{2^n} - \frac{H(2^{n+1} x)}{2^{n+1}} \right\| \leq |2|^{-(n+1)}\omega(2^n x, 2^n x, 2^n x) \tag{51}$$

It follows from (44) and (51) that the sequence $\left\{ \frac{H(2^n x)}{2^n} \right\}_{n \geq 1}$ is convergent. Set $\Re(x) := \lim_{n\to\infty} \frac{H(2^n x)}{2^n}$. On the other hand, it follows from (51) that

$$\begin{aligned} \left\| \frac{H(2^p x)}{2^p} - \frac{H(2^q x)}{2^q} \right\| &\leq max\left\{ \left\| \frac{H(2^k x)}{2^k} - \frac{H(2^{k+1} x)}{2^{k+1}} \right\| ;\ p \leq k < q \right\} \\ &\leq \frac{1}{|2|} max\left\{ |2|^{-k}\omega(2^k x, 2^k x, 2^k x); p \leq k < q \right\} \end{aligned}$$

for all $x \in G$ and all non-negative integers p, q with $q > p \geq 0$. Letting $p = 0$ and passing the limit $q \to \infty$ in the last inequality and using (45), we obtain (48). The rest of the proof is similar to the proof of Theorem (6).

Corollary 9. *Let $\xi : [0, \infty) \to [0, \infty)$ be a function satisfying*

$$\xi(|2|t) \leq \xi(|2|)\xi(t) \quad (t \geq 0), \quad \xi(|2|) < |2| \tag{52}$$

Let $\kappa > 0$ and $f : G \to X$ be a mapping satisfying

$$\left\| H\left(\frac{x+y}{2} + z\right) - \frac{H(x) + H(y)}{2} - H(z) \right\|_Y \leq \kappa(\xi(|x|).\xi(|y|).\xi(|z|)). \tag{53}$$

for all $x, y, z \in G$. Then, there exists a unique additive mapping $\Im : G \to X$ such that

$$||Q(x) - \Im(x)|| \leq \frac{\kappa\xi^3(|x|)}{|2|} \tag{54}$$

Proof. Define $\zeta : G^3 \to [0, \infty)$ by $\zeta(x, y, z) := \kappa(\xi(|x|).\xi(|y|).\xi(|z|))$ and apply Theorem (8) to get the result.

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