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TRACKING PROBLEM FOR LINEAR PERIODIC, DISCRETE-TIME STOCHASTIC SYSTEMS IN HILBERT SPACES

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Abstract. The aim of this paper is to solve the tracking problem for linear periodic discrete-time systems with independent random perturbations, in Hilbert spaces. Under stabilizability conditions, we will find an optimal control, which minimize the cost function associated to this problem, in the case when the control weight cost is only nonnegative and not necessarily uniformly positive.

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1.Notations and the statement of the problem

Throughout this paper the spaces H, V, U are separable real Hilbert spaces. We will denote by $L(H,V)$ (respectively $L(H)$) the Banach space of all bounded linear operators which transform H into V (respectively H). We write $\langle .,. \rangle$ for the inner product and $\|.\|$ for norms of elements and operators. If $A \in L(H)$ then A^* is the adjoint operator of A. The operator $A \in L(H)$ is said to be nonnegative and we write $A \geq 0$, if A is self-adjoint and $\langle Ax, x \rangle \geq 0$ for all $x \in H$. For every Hilbert space H we will denote by $\mathcal{H}^s$ the Banach subspace of $L(H)$ formed by all self-adjoint operators, by $\mathcal{H}^+$ the cone of all nonnegative operators of $\mathcal{H}^s$ and by I the identity operator on H. The operator $A \in \mathcal{H}^+$ is positive (and we write $A > 0$) if A is invertible. The sequence $L_n \in L(H,V), n \in \mathbf{Z}$ is *bounded* on $\mathbf{Z}$ if $\sup_{n \in \mathbf{Z}} \|L_n\| < \infty$ and is τ *-periodic* if $L_n = L_{n+\tau}$ for all $n \in \mathbf{N}$. We say that the sequence $L_n \in \mathcal{H}^s, n \in \mathbf{N}$ is uniformly positive if there exists $a > 0$ such that $L_n \geq aI$ for all $n \in \mathbf{N}$.

Let $(\Omega, \mathcal{F}, P)$ be a probability space and ξ be a real valued random variable on Ω. If ξ is a real or H -valued random variable on Ω, we write $E(\xi)$ for mean value (expectation) of ξ and we will denote by $L^2(\Omega, \mathcal{F}, P, H)$ or $L^2(H)$ the Hilbert space of all equivalence class of H-valued random variables η such that $E\left\|\eta\right\|^2 < \infty$.

Let $\xi_n \in L^2(\mathbf{R}), n \in \mathbf{Z}$ be real and independent random variables, which satisfy the conditions $E(\xi_n) = 0$ and let $\mathcal{F}_n$, $n \in \mathbf{Z}$ be the $\sigma-$ algebra generated by $\{\xi_i, i \leq n-1\}$. Let $\widetilde{U}_k$ be the set of all sequences $\{u_n\}_{n \geq k}$, where u_n is an U-valued random variable, $\mathcal{F}_n-$ measurable with the property $\sup\limits_{n\geq k} E\left\|u_n\right\|^2 < \infty$.

We consider the system with control, denoted $\{A : D, B : H\}$

$$\begin{aligned} x_{n+1} &= A_n x_n + \xi_n B_n x_n + (D_n + \xi_n H_n)\, u_n \\ x_k &= x \in H, k \in \mathbf{Z} \end{aligned} \tag{1}$$

and the output

$$y_n = x_n + P_n u_n \tag{2}$$

where $A_n, B_n \in L(H)$, $D_n, H_n \in L(U, H), P_n \in L(U, V)$ for all $n \in \mathbf{Z}$, $n \geq k$ and the control $u = \{u_k, u_{k+1}, ...\}$ belongs to the class $\widetilde{U}_k$. If one of D_n or H_n are missing we will remove it from the notation $\{A : D, B : H\}$(e.g. $\{A, B\}$ if $D = H = 0$).

Throughout this paper we assume the following hypothesis:

H_0: The sequences A_n, $B_n \in L(H)$, $D_n, H_n \in L(U, H), P_n \in L(U, H)$, $K_n \in \mathcal{U}^+, r_n \in H$ and $b_n \in \mathbf{R}$,$n \in \mathbf{Z}$ are τ-periodic, $\tau \in N^*$ and

$$D_n^* D_n + b_n H_n^* H_n \geq \delta I, \delta > 0 \text{ for all } n \in \{0, .., \tau - 1\}. \tag{3}$$

The tracking problem consist in finding a feedback control u in a suitable class of controls such us the solution x_n of the controlled system (1) is "as close as possible" to a given, bounded signal $r = \{r_n\}$.

For every $x \in H$ and $k \in \mathbf{Z}$, we look for an optimal control $u \in U_{k,x}$, which minimize the following cost functional

$$I_k(x, u) = \overline{\lim_{q\to\infty}} \frac{1}{q - k} E \sum_{n=k}^{q-1} [\left\|x_n - r_n\right\|^2 + < K_n u_n, u_n >], \tag{4}$$

where x_n is the solution of (1) and $u \in U_{k,x}$ ($U_{k,x} \subset \widetilde{U}_k$ is the subset of all admissible controls with the property that (1) has a bounded solution). Now it is clear that if $u \in U_{k,x}$ then $I_k(x, u) < \infty$.

Under stabilizability conditions (see Theorem 12) we will design the optimal control, which minimize the functional cost $I_k(x,u)$. We note that in most of the previous work (see [2], [5], [7] and the references therein) the control weight K_n is positive definite for the well posedness of the problem. Recently works (see [1] for the continuous case) show that some stochastic quadratic control problems (and consequently some tracking problems with reduces to such a problems) with $H_n \neq 0$ are nontrivial even for $K_n < 0$. For this reason we consider in this paper the control weight K_n belonging to the largest class $\mathcal{U}^+$ and this choice is compenssed by a quadratic term, which is related to H_n(see (3)).

2.Bounded solutions of the affine discrete time systems

Let us denote by $X(n,k)$ $n \geq k \geq 0$, the random evolution operator associated to $\{A,B\}$ that is $X(k,k) = I$ and $X(n,k) = (A_{n-1} + \xi_{n-1}B_{n-1})...(A_k + \xi_k B_k)$, for all $\dot{n} > k$. Then it is known that the linear discrete time system $\{A,B\}$ with the initial condition $x_k = x \in H$ has a unique solution $x_n = x_n(k,x)$ given by $x_n = X(n,k)x$.

Definition 1. *We say that $\{A,B\}$ is uniformly exponentially stable iff there exist $\beta \geq 1$, $a \in (0,1)$ such that we have*

$$E\left\|X(n,k)x\right\|^2 \leq \beta a^{n-k}\left\|x\right\|^2$$

for all $n \geq k \geq n_0$ and $x \in H$.

Remark 2.*If $B_n = 0$ for all $n \in \mathbf{Z}$, we obtain the definition of the uniform exponential stability of the deterministic system $x_{n+1} = A_n x_n, x_k = x \in H, n \geq k$ denoted $\{A\}$.*

Proposition 3. (see [8], or [2] for the finite dimensional case) *If $g_n \in H, n \in \mathbf{Z}$ is a bounded sequence and $\{A\}$ is uniformly exponentially stable then the system*

$$y_n = A_n^* y_{n+1} + g_n \tag{5}$$

has a unique bounded on $\mathbf{Z}$ solution. Moreover, if H_1 holds and g_n is τ-periodic, then y_n is τ-periodic.

Definition 4. *A sequence $\{\eta_n\}, n \in \mathbf{Z}$ of H-valued random variables is τ-periodic, $\tau \in N^*$ if*

$$P\{\eta_{n_1+\tau} \in A_1, ..., \eta_{n_m+\tau} \in A_m\} = P\{\eta_{n_1} \in A_1, ..., \eta_{n_m} \in A_m\}, \tag{6}$$

for all $n_1, n_2, ..., n_m \in \mathbf{Z}$ *and all* $A_p \in \mathcal{B}(H)$, $p = 1, .., m$.

Reasoning as in [8], we can establish the following result:

PROPOSITION 5. *Assume that the sequences* D_n, H_n, b_n *and* $q_n \in H$ *are bounded on* $\mathbf{Z}$. *If* $\{A, B\}$ *is uniformly exponentially stable then the system*

$$x_{n+1} = A_n x_n + \xi_n B_n x_n + (D_n + \xi_n H_n)\, q_n \tag{7}$$

(without initial condition) has a unique solution in $L^2(H)$ *which is mean square bounded on* $\mathbf{Z}$, *that is there exists* $M > 0$ *such that* $E\|x_n\|^2 < M$ *for all* $n \in \mathbf{Z}$. *Moreover, if* H_1 *is satisfied and the sequences* $q_n, \{\xi_n\}, n \in \mathbf{Z}$ *are* τ*-periodic, then the unique solution of (7) is* $\tau-$*periodic.*

3. Discrete-time Riccati equation of stochastic control

We consider the mappings

$$\begin{aligned}
&\mathcal{D}_n : \mathcal{H}^s \rightarrow \mathcal{U}^s, \mathcal{D}_n(S) = D_n^* S D_n + b_n H_n^* S H_n,\\
&\mathcal{V}_n : \mathcal{H}^s \rightarrow L(H, U), \mathcal{V}_n(S) = D_n^* S A_n + b_n H_n^* S B_n\\
&\mathcal{A}_n : \mathcal{H}^s \rightarrow \mathcal{H}^s, \mathcal{A}_n(S) = A_n^* S A_n + b_n B_n^* S B_n
\end{aligned}$$

and we define the transformation

$$\begin{aligned}
&\mathcal{G}_n(S) = (\mathcal{V}_n(S))^* (K_n + \mathcal{D}(S))^{-1} \mathcal{V}_n(S), S > 0,\\
&\mathcal{G}_n(0) = 0
\end{aligned}$$

It is easy to see that if $S > 0$, then it follows, by (3) that $K_n + \mathcal{D}(S)$ is invertible. We introduce the following Riccati equation

$$R_n = \mathcal{A}_n(R_{n+1}) + I - \mathcal{G}_n(R_{n+1}) \tag{8}$$

$$R_n > 0, n \in \mathbf{Z}, \tag{9}$$

connected with the quadratic cost (4).

DEFINITION 6. *A sequence* $\{R_n\}_{n\in\mathbf{Z}}$, $R_n > 0$ *such as (8) holds is said to be a solution of the Riccati equation (8).*

Let us consider the sequence $R(M,M)=0$*, we will prove that the following sequence*

$$R(M,n)=\mathcal{A}_n(R(M,n+1))+I-\mathcal{G}_n(R(M,n+1)) \tag{10}$$

is well defined for all $n\leq M-1$*.*

LEMMA 7. *The sequence* $R(M,n)$ *has the following properties*
a) $R(M,n)>I, R(M+\tau,n+\tau)=R(M,n)$
b) $R(M-1,n)\leq R(M,n)$.
for all $n\leq M-1$

Proof. a) Let us denote

$$F(M,n)=-\left[K_n+\mathcal{D}_n(R(M,n+1))\right]^{-1}\mathcal{V}_n(R(M,n+1)), n\in\mathbf{N}^*.$$

Then (10) can be written

$$\begin{aligned} R(M,n)=&\left[A_n^*+F^*(M,n)D_n^*\right]R(M,n+1)\left[A_n+D_nF(M,n)\right]+\\ &b_n\left[B_n^*+F^*(M,n)H_n^*\right]R(M,n+1)\left[B_n+H_nF(M,n)\right]+I\\ &+F^*(M,n)K_nF^*(M,n)\end{aligned}$$

and it is clear that $R(M,n)\geq I$ *for all* $M>n$*. Using* H_0 *we obtain* $R(M+\tau,n+\tau)=R(M,n)$ *and arguing as in the proof of the Lemma 3 from [7] it follows b).*

DEFINITION 8.*[3] The system (1) is stabilizable if there exists a bounded on* $\mathbf{Z}$ *sequence* $F=\{F_n\}_{n\in\mathbf{Z}}, F_n\in L(H,U)$ *such that* $\{A+DF,B+HF\}$ *is uniformly exponentially stable.*

DEFINITION 9.*[3]A solution* $R=(R_n)_{n\in\mathbf{Z}}$ *of (8) is said to be stabilizing for (1) if* $\{A+DF,B+HF\}$ *with*

$$F_n=-(K_n+\mathcal{D}_n(R_{n+1}))^{-1}\mathcal{V}_n(R_{n+1}), n\in\mathbf{Z} \tag{11}$$

is uniformly exponentially stable.

THEOREM 10. *Suppose (1) is stabilizable. Then the Riccati equation (8) admits a nonnegative* τ*- periodic solution. Moreover, this solution is stabilizing for (1).*

Proof. Since (1) is stabilizable it follows that there exists a bounded on N^* sequence $F=\{F_n\}_{n\in\mathbf{N}^*}$, $F_n\in L(H,U)$ such that $\{A+DF,B+HF\}$ is uniformly exponentially stable.

Let x_n be the solution of $\{A+DF, B+HF\}$ with the initial condition $x_k = x$ and let us consider $\overline{u}_n = F_n x_n$.

Since F_n is bounded on N^*, it is not difficult to see that $\overline{u}_n \in \widetilde{U}_{k,x}$. As in the proof of Proposition 3 in [7] it follows that there exists the positive constant λ such as

$$0 \leq \langle R(M-1,k)x, x\rangle \leq \langle R(M,k)x, x\rangle \leq V(M,k,x,\overline{u}) \leq \lambda \left\|x\right\|^2 .$$

where $R(M,n)$ is the solution of the Riccati equation (8) with the final condition $R(M,M)=0$. Thus, there exists $R_k \in L(H)$ such that $0 \leq R(M,k) \leq R_k \leq \lambda I$ for $M \in \mathbf{N}^*$, $M \geq k$ and $R(M,k) \underset{M\to\infty}{\to} R_k$ in the strong operator topology. We denote

$$L = \lim_{M\to\infty}(< \mathcal{G}_n(R(M,n+1))x, x > - < \mathcal{G}_n(R(n+1))x, x >)$$

, $P_{M,n} = K_n + \mathcal{D}_n\left(R(M,n+1)\right)$, $P_n = K_n + \mathcal{D}_n\left(R\left(n+1\right)\right) Y_n = \mathcal{V}_n\left(R\left(n+1\right)\right)$ and $Y_{M,n} = \mathcal{V}_n\left(R\left(M,n+1\right)\right)$. From the definition of $\mathcal{G}_n$ we get

$$\begin{aligned} L &= \lim_{M\to\infty}(\left\langle P_{M,n}^{-1} Y_{M,n}x, Y_{M,n}x\right\rangle - \left\langle P_n^{-1} Y_n x, Y_n x\right\rangle) \\ &= \lim_{M\to\infty}(\left\langle \left(P_{M,n}^{-1} - P_n^{-1}\right) Y_n x, Y_n x\right\rangle \\ &+ \left\langle P_{M,n}^{-1}\left(Y_{M,n}x - Y_n x\right), \left(Y_{M,n}x - Y_n x\right)\right\rangle + 2\left\langle P_{M,n}^{-1} Y_n x, \left(Y_{M,n}x - Y_n x\right)\right\rangle). \end{aligned}$$

Using Lemma 7 it follows that $P_{M,n} \geq \mathcal{D}_n\left(R(M,n+1)\right) \geq \delta I$, $\delta > 0$ and we deduce that $\left\|P_{M,n}^{-1}\right\| \leq \frac{1}{\delta}$ for all $M \geq n+1 \geq k$. Thus

$$\begin{aligned} \lim_{M\to\infty}\left\|P_{M,n}^{-1}x - P_n^{-1}x\right\| &\leq \lim_{M\to\infty}\left\|P_{M,n}^{-1}\right\|\left\|P_{M,n}u - P_n u\right\| \\ &\leq \frac{1}{\delta}\lim_{M\to\infty}\left\|P_{M,n}u - P_n u\right\| = 0 \end{aligned} \tag{12}$$

where $u = P_n^{-1}x$. Now it is a simple exercise to prove that $L=0$ and

$$\lim_{M\to\infty}\left\langle \mathcal{G}_n(R(M,n+1))x, x\right\rangle = \left\langle \mathcal{G}_n(R(n+1))x, x\right\rangle .$$

From the definition of $R(M,n)$ and the above result we deduce that R_n is a nonnegative, bounded on $\mathbf{Z}$ solution of (8). From the statement a) of Lemma 7 it follows that R_n is τ- periodic.

Before to prove the last statement, we will see that the Riccati equation (8) is equivalent with the following equation

$$R_n = (A_n^* + F_n^* D_n^*) R_{n+1} (A_n + D_n F_n) +$$
$$b_n (B_n^* + F_n^* H_n^*) R_{n+1} (B_n + H_n F_n) + I + F_n^* K_n F_n^*,$$

where F_n is given by (11). If we denote $\widetilde{W}_n = I + F_n^* K_n F_n^* \geq I$, it is clear that the Lyapunov equation

$$L_n = (A_n^* + F_n^* D_n^*) L_{n+1} (A_n + D_n F_n) +$$
$$b_n (B_n^* + F_n^* H_n^*) R_{n+1} (B_n + H_n F_n) + \widetilde{W}_n,$$

has a nonnegative bounded on $\mathbf{Z}$ and uniformly positive ($L_n > I$) solution, namely R_n. It follows (see [6]) that $\{A + DF, B + HF\}$ is uniformly exponentially stable. Hence the nonnegative τ- periodic solution of the Riccati equation is stabilizing for (1).

Remark 11. *As in [7] it can be proved that the Riccati equation considered in this paper has at most one stabilizing solution. Thus, it follows that, under the hypotheses of the above theorem, the Riccati equation has a unique nonnegative and τ- periodic solution.*

4. The main results

Let us denote $f_n = A_n r_n - r_{n+1}$ and $p_n = B_n r_n$. The following theorem gives the optimal control, which minimize the cost function (4).

Theorem 12. *Assume that the hypotheses of the Theorem 10 hold. Let R_n be the unique τ-periodic solution of the Riccati equation (8). If g_n and h_n are the unique τ-periodic solutions of the Lyapunov equations*

$$g_n = (A_n + D_n F_n)^* g_{n+1} + R_n f_{n-1} \tag{13}$$

$$h_n = (A_n + D_n F_n)^* h_{n+1} + b_n (B_n + H_n F_n)^* R_{n+1} p_n \tag{14}$$

where F_n is given by (11), then

$$I(\overline{u}) = \frac{1}{\tau} \sum_{i=1}^{\tau} - \left\| V_i^{-1/2} \left[D_i^* (g_{i+1} + h_{i+1}) + b_i H_i^* R_{i+1} p_i \right] \right\|^2 \tag{15}$$
$$- \langle R_{i+1} f_i, f_i \rangle + b_i \langle R_{i+1} p_i, p_i \rangle + 2 \langle g_{i+1} + h_{i+1}, f_i \rangle]$$

where the optimal control is

$$\overline{u}_n = F_n \overline{x}_n - F_n r_n - \left[D_n^* \left(g_{n+1} + h_{n+1} \right) + b_n H_n^* R_{n+1} p_n \right], \tag{16}$$

$n \geq k$, $\overline{x}_n$ is the corresponding solution of the system (1) and $V_n = K_n + \mathcal{D}_n(R_{n+1})$.

PROOF. The equations (13) and (14) have unique solutions according Proposition 3, since the solution of the Riccati equation (8) is stabilizing for (1). Let us consider the function

$$v_n : H \to R, v_n(x) = \langle R_n x, x \rangle + 2E \langle g_n - R_n f_{n-1} + h_n, x \rangle .$$

If x_n is the solution of the system (1), then

$$\begin{aligned} Ev_{n+1}\left(x_{n+1} - r_{n+1} \right) &= Ev_n \left(x_n - r_n \right) - E[\left\| x_n - r_n \right\|^2 + \langle K_n u_n, u_n \rangle] \qquad (17) \\ +E \left\| V_n^{1/2} \left(F_n \left(x_n - r_n \right) - u_n - V_n^{-1} \left[D_n^* \left(g_{n+1} + h_{n+1} \right) + b_n H_n^* R_{n+1} p_n \right] \right) \right\|^2 \\ - \left\| V_n^{-1/2} \left[D_n^* \left(g_{n+1} + h_{n+1} \right) + b_n H_n^* R_{n+1} p_n \right] \right\|^2 \\ - \langle R_{n+1} f_n, f_n \rangle + b_n \langle R_{n+1} p_n, p_n \rangle + 2 \langle g_{n+1} + h_{n+1}, f_n \rangle . \end{aligned}$$

Let $\overline{x}_n$ be the solution of the system (1), where

$$\overline{u}_n = F_n \overline{x}_n - F_n r_n - V_n^{-1} \left[D_n^* \left(g_{n+1} + h_{n+1} \right) + b_n H_n^* R_{n+1} p_n \right].$$

It is not difficult to see that $\overline{x}_n$ and $\overline{u}_n$ are bounded on $\{n \in \mathbf{N}, n \geq k\}$ and $\overline{u} \in U_{k,x}$. Using (17) we get

$$\begin{gathered} \frac{1}{n-k} [v_k(x - r_k) - Ev_{n+1} \left(\overline{x}_{n+1} - r_{n+1} \right)] = \qquad (18) \\ \frac{1}{n-k} \sum_{i=k}^{n-1} E[\left\| \overline{x}_i - r_i \right\|^2 + \langle K \overline{u}_i, \overline{u}_i \rangle] + \\ \left\| V_i^{-1/2} \left[D_i^* \left(g_{i+1} + h_{i+1} \right) + b_i H_i^* R_{i+1} p_i \right] \right\|^2 + \\ \langle R_{i+1} f_i, f_i \rangle - b_i \langle R_{i+1} p_i, p_i \rangle - 2 \langle g_{i+1} + h_{i+1}, f_i \rangle . \end{gathered}$$

Since r_n is bounded on $\mathbf{Z}$ and R_n is stabilizing we deduce that there exists

$P > 0$ such that $Ev_{n+1}(\overline{x}_{n+1} - r_{n+1}) \leq P$. As $n \to \infty$ in (18), it follows

$$I_k(x, \overline{u}) = \overline{\lim_{n\to\infty}} \frac{1}{n-k} \sum_{i=k}^{n-1} [- \left\| V_i^{-1/2} \left[D_i^* \left(g_{i+1} + h_{i+1} \right) + b_i H_i^* R_{i+1} p_i \right] \right\|^2$$
$$- \left\langle R_{i+1} f_i, f_i \right\rangle + b_i \left\langle R_{i+1} p_i, p_i \right\rangle + 2 \left\langle g_{i+1} + h_{i+1}, f_i \right\rangle]$$
$$= \frac{1}{\tau} \sum_{i=1}^{\tau} [- \left\| V_i^{-1/2} \left[D_i^* \left(g_{i+1} + h_{i+1} \right) + b_i H_i^* R_{i+1} p_i \right] \right\|^2$$
$$- \left\langle R_{i+1} f_i, f_i \right\rangle + b_i \left\langle R_{i+1} p_i, p_i \right\rangle + 2 \left\langle g_{i+1} + h_{i+1}, f_i \right\rangle]$$

If $u \in U_{k,x}$, it is not difficult to deduce from (17), that $I_k(x, \overline{u}) \leq I_k(x, u)$. Thus $\min_{u \in U_{k,x}} I_k(x, u) = I_k(x, \overline{u})$.

Remark 13. *Assume that the hypotheses of the above theorem fulfilled. If $\overline{u}_n$ is given by (16), then (1) has a unique bounded solution on $\mathbf{Z}$, according Proposition 5. Denoting by $\overline{X}(n,k)$ the random evolution operator associated to the system $\{A + DF, B + HF\}$, we get*

$$\overline{x}_n = - \sum_{i=-\infty}^{n-1} \overline{X}(n, i+1) \left(D_i + \xi_i H_i \right) \tag{19}$$
$$\{ V_i^{-1} \left[D_i^* \left(g_{i+1} + h_{i+1} \right) + b_i H_i^* R_{i+1} p_i \right] + F_i r_i \}$$

Since the optimal cost doesn't depend on the initial value, x_k, it is not difficult to see that, if we use the above solution in (16), we obtain the optimal control, which minimize the cost function (4).

Moreover, if $\{\xi_n\}, n \in \mathbf{Z}$ is τ-periodic, then $\overline{x}_n$ is τ-periodic (19).

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