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THE SIMPLEST EQUATION METHOD FOR SOLVING SOME IMPORTANT NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. The simplest equation method presents a wide applicability to handling nonlinear wave equations. In this paper, we establish travelling wave solutions for some nonlinear evolution equations. The simplest equation method is used to construct the travelling wave solutions of new Hamiltonian amplitude equation, $(3+1)$-dimensional generalized KP equation, Burgers-KP equation, coupled Higgs field equation, generalized Zakharov System. New Hamiltonian amplitude equation is an equation which governs certain instabilities of modulated wave trains, with the additional term $-\epsilon u_{xt}$ overcoming the ill-posedness of the unstable nonlinear Schrödinger equation. It is a Hamiltonian analogue of the Kuramoto-Sivashinski equation which arises in dissipative systems and is apparently not integrable.

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1. Introduction

Nonlinear phenomena play crucial roles in applied mathematics and physics. Calculating exact and numerical solutions, in particular the traveling wave solutions of nonlinear equations in mathematical physics, plays an important role in soliton theory. Recently many new approaches for finding the exact solutions to nonlinear equations have been proposed, such as ansatz method and topological solitons [1-4], tanh method [5,6], multiple exp-function method [7], simplest equation method [8-11], Hirotas direct method [12,13], transformed rational function method [14].
Using simplest equation method in work [15] exact solutions of the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity, the nonlinear Schrödinger's equation were obtained.
The paper is arranged as follows. In Section 2, we describe briefly the simplest equation method. In Sections 3-7, we apply this method to new Hamiltonian amplitude equation, $(3+1)$-dimensional generalized KP equation, Burgers-KP equation, coupled Higgs field equation and generalized Zakharov System.

2. The simplest equation method

Step 1. We first consider a general form of nonlinear equation

$$P(u, u_x, u_t, u_{xx}, u_{xt}, ...) = 0. \tag{1}$$

Step 2. To find the traveling wave solution of Eq. (1) we introduce the wave variable $\xi = x - ct$ so that

$$u(x,t) = y(\xi). \tag{2}$$

Based on this we use the following changes

$$\begin{aligned} \frac{\partial}{\partial t}(.) &= -c\frac{\partial}{\partial \xi}(.), \\ \frac{\partial}{\partial x}(.) &= \frac{\partial}{\partial \xi}(.), \\ \frac{\partial^2}{\partial x^2}(.) &= \frac{\partial^2}{\partial \xi^2}(.) \end{aligned} \tag{3}$$

and so on for other derivatives.
Using (3) changes the PDE (1) to an ODE

$$Q(y, \frac{\partial y}{\partial \xi}, \frac{\partial^2 y}{\partial \xi^2}, ...) = 0, \tag{4}$$

where $y = y(\xi)$ is an unknown function, Q is a polynomial in the variable y and its derivatives.
Step 3. The basic idea of the simplest equation method consists in expanding the solutions $y(\xi)$ of Eq. (4) in a finite series

$$y(\xi) = \sum_{i=0}^{l} a_i z^i, \quad a_l \neq 0, \tag{5}$$

where the coefficients a_i are independent of ξ and $z = z(\xi)$ are the functions that satisfy some ordinary differential equations.
In this paper, we use the Bernoulli equation [16] as simplest equation

$$\frac{dz}{d\xi} = az(\xi) + bz^2(\xi), \tag{6}$$

Eq. (6) admits the following exact solutions

$$z(\xi) = \frac{a\exp[a(\xi + \xi_0)]}{1 - b\exp[a(\xi + \xi_0)]}, \tag{7}$$

for the case $a > 0, b < 0$ and

$$z(\xi) = -\frac{a\exp[a(\xi+\xi_0)]}{1+b\exp[a(\xi+\xi_0)]}, \tag{8}$$

for the case $a < 0, b > 0$, where ξ_0 is a constant of integration.
Remark 1. l is a positive integer, in most cases, that will be determined. To determine the parameter l, we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms.
Step 4. Substituting (5) into (4) with (6), then the left hand side of Eq. (4) is converted into a polynomial in $z(\xi)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, a, b, c.
Step 5. Solving the algebraic equations obtained in step 4, and substituting the results into (5), then we obtain the exact traveling wave solutions for Eq. (1).

Remark 2. In Eq. (6), when $a = A$ and $b = -1$ we obtain the Bernoulli equation

$$\frac{dz}{d\xi} = Az(\xi) - z^2(\xi). \tag{9}$$

Eq. (9) admits the following exact solutions

$$z(\xi) = \frac{A}{2}[1+\tanh(\frac{A}{2}(\xi+\xi_0))], \tag{10}$$

when $A > 0$, and

$$z(\xi) = \frac{A}{2}[1-\tanh(\frac{A}{2}(\xi+\xi_0))], \tag{11}$$

when $A < 0$.
Remark 3. This method is a simple case of the Ma- Fuchssteiner method [16].

3. New Hamiltonian amplitude equation

A new Hamiltonian amplitude equation

$$iu_x + u_{tt} + 2\sigma|u|^2u - \varepsilon u_{xt} = 0, \tag{12}$$

where $\sigma = \pm 1$, $\varepsilon << 1$, was recently introduced by Wadati et al., [17].
By make the transformation

$$u(x,t) = e^{i(\alpha x+\beta t)}y(\xi), \quad \xi = ik(x-\lambda t), \quad \lambda = \frac{1-\varepsilon\beta}{2\beta-\varepsilon\alpha}, \tag{13}$$

Eq. (12) becomes

$$-k^2(\lambda^2+\varepsilon\lambda)y_{\xi\xi} - (\alpha+\beta^2-\varepsilon\beta\alpha)y + 2\sigma y^3 = 0. \tag{14}$$

For the solutions of Eq. (14), we make the following ansatz

$$y(\xi) = \sum_{i=0}^{l} a_i z^i, \quad a_l \neq 0, \tag{15}$$

where a_i are all real constants to be determined, l is a positive integer which can be determined by balancing the highest order derivative term with the highest order nonlinear term after substituting ansatz (15) into Eq. (14), where z satisfies Eq. (6).
When balancing $y_{\xi\xi}$ with y^3 then gives $l + 2 = 3l \Longrightarrow l = 1$. Therefore, we may choose

$$y(\xi) = a_0 + a_1 z(\xi). \tag{16}$$

Substituting (16) along with (6) in Eq. (14) and then setting the coefficients of $z^j (j = 3, 2, 1, 0)$ to zero in the resultant expression, we obtain a set of algebraic equations involving a_0, a_1, a, b, α and β as

$$\begin{gathered} -2k^2(\lambda^2 + \varepsilon\lambda)b^2 a_1 + 2\sigma a_1^3 = 0, \\ -3k^2(\lambda^2 + \varepsilon\lambda)aba_1 + 6\sigma a_0 a_1^2 = 0, \\ -k^2(\lambda^2 + \varepsilon\lambda)a^2 a_1 + 6\sigma a_0^2 a_1 - (\alpha + \beta^2 - \varepsilon\beta\alpha)a_1 = 0, \\ 2\sigma a_0^3 - (\alpha + \beta^2 - \varepsilon\beta\alpha)a_0 = 0. \end{gathered} \tag{17}$$

With the aid of Maple, we shall find the special solution of the above system

$$a_0 = \pm\sqrt{\frac{\alpha + \beta^2 - \varepsilon\beta\alpha}{2\sigma}}, \quad a = \pm\sqrt{\frac{2(\alpha + \beta^2 - \varepsilon\beta\alpha)}{k^2\lambda(\lambda + \varepsilon)}}, \quad b = \pm\sqrt{\frac{\sigma}{k^2\lambda(\lambda + \varepsilon)}}a_1, \tag{18}$$

where α, β and a_1 are arbitrary constants.
Assuming $a > 0$ and choosing $b < 0$. Therefore, using solution (7) of Eq. (6), ansatz (16) , we obtain the following traveling-wave solution of Eq. (14)

$$y(\xi) = \pm\sqrt{\frac{\alpha + \beta^2 - \varepsilon\beta\alpha}{2\sigma}}(1 + \frac{2a_1\sqrt{\sigma}\exp[\sqrt{\frac{2(\alpha+\beta^2-\varepsilon\beta\alpha)}{k^2\lambda(\lambda+\varepsilon)}}(\xi + \xi_0)]}{k\sqrt{\lambda(\lambda + \varepsilon)} - \sqrt{\sigma}a_1\exp[\sqrt{\frac{2(\alpha+\beta^2-\varepsilon\beta\alpha)}{k^2\lambda(\lambda+\varepsilon)}}(\xi + \xi_0)]}). \tag{19}$$

Then the exact solution to Eq. (12) can be written as

$$u(x,t) = \pm\sqrt{\frac{\alpha + \beta^2 - \varepsilon\beta\alpha}{2\sigma}}(1 + \frac{2a_1\sqrt{\sigma}e^{\sqrt{\frac{2(\alpha+\beta^2-\varepsilon\beta\alpha)}{k^2\lambda(\lambda+\varepsilon)}}(ik(x-\lambda t)+\xi_0)}}{k\sqrt{\lambda(\lambda + \varepsilon)} - \sqrt{\sigma}a_1 e^{\sqrt{\frac{2(\alpha+\beta^2-\varepsilon\beta\alpha)}{k^2\lambda(\lambda+\varepsilon)}}(ik(x-\lambda t)+\xi_0)}})e^{i(\alpha x + \beta t)}, \tag{20}$$

where $\lambda = \frac{1-\varepsilon\beta}{2\beta-\varepsilon\alpha}$.
Substituting (16) along with (9) in Eq. (14) and setting all the coefficients of powers z to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$a_0 = \pm\sqrt{\frac{\alpha+\beta^2-\varepsilon\beta\alpha}{2\sigma}}, \quad a_1 = \pm k\sqrt{\frac{\lambda(\lambda+\varepsilon)}{\sigma}}, \quad A = -\sqrt{\frac{2(\alpha+\beta^2-\varepsilon\beta\alpha)}{k^2\lambda(\lambda+\varepsilon)}}, \tag{21}$$

where β, α and k are arbitrary constants.
Therefore, using solution (11) of Eq. (9), ansatz (16), we obtain the following exact solution of Eq. (14)

$$y(\xi) = \pm\sqrt{\frac{\alpha+\beta^2-\varepsilon\beta\alpha}{2\sigma}}\tanh[\sqrt{\frac{\alpha+\beta^2-\varepsilon\beta\alpha}{2k^2\lambda(\lambda+\varepsilon)}}(\xi+\xi_0)]. \tag{22}$$

Then, the exact solution to Eq. (12) can be written as

$$u(x,t) = \pm\sqrt{\frac{\alpha+\beta^2-\varepsilon\beta\alpha}{2\sigma}}\tanh[\sqrt{\frac{\alpha+\beta^2-\varepsilon\beta\alpha}{2k^2\lambda(\lambda+\varepsilon)}}(ik(x-\lambda t)+\xi_0)]e^{i(\alpha x+\beta t)}, \tag{23}$$

where $\lambda = \frac{1-\varepsilon\beta}{2\beta-\varepsilon\alpha}$.

4. (3 + 1)-dimensional generalized KP equation

Let us consider the (3 + 1)-dimensional generalized KP equation [18, 19]

$$u_{xxxy} + 3(u_x u_y)_x + u_{tx} + u_{ty} - u_{zz} = 0. \tag{24}$$

We use the wave transformation

$$u(x,y,z,t) = y(\xi), \quad \xi = kx + \alpha y + \beta z - \gamma t, \tag{25}$$

where k, α, β and γ are constants, all of them are to be determined.
Substituting (25) into (24), we obtain ordinary differential equation:

$$\alpha k^3 y_{\xi\xi\xi\xi} + 6k^2\alpha y_\xi y_{\xi\xi} - (k\gamma + \alpha\gamma + \beta^2)y_{\xi\xi} = 0. \tag{26}$$

When balancing $y_{\xi\xi\xi\xi}$ with $y_\xi y_{\xi\xi}$ then gives $l+4 = l+1+l+2 \Longrightarrow l = 1$. Therefore, we may choose

$$y(\xi) = a_0 + a_1 z(\xi). \tag{27}$$

Substituting (27) along with (6) in Eq. (26) and then setting the coefficients of $z^j (j = 5,4,3,2,1)$ to zero in the resultant expression, we obtain a set of algebraic equations involving $a_0, a_1, a, b, k, \alpha, \beta$ and γ as

$$24\alpha k^3 a_1 b^4 + 12k^2\alpha a_1^2 b^3 = 0, \tag{28}$$

$$60\alpha k^3b^3aa_1 + 30k^2\alpha ab^2a_1^2 = 0,$$
$$-2(k\gamma + \alpha\gamma + \beta^2)a_1b^2 + 50\alpha k^3b^2a^2a_1 + 24k^2\alpha a^2ba_1^2 = 0,$$
$$-3(k\gamma + \alpha\gamma + \beta^2)aba_1 + 15\alpha k^3a_1ba^3 + 6k^2\alpha a^3a_1^2 = 0,$$
$$\alpha k^3a^4a_1 - (k\gamma + \alpha\gamma + \beta^2)a^2a_1 = 0.$$

With the aid of Maple, we shall find the special solution of the above system

$$a_1 = -2kb, \quad \gamma = \frac{k^3a^2\alpha - \beta^2}{k+\alpha}, \tag{29}$$

where a_0, a, b, k, α and β are arbitrary constants.
Assuming $a > 0$ and choosing $b < 0$. Therefore, using solution (7) of Eq. (6), ansatz (27) , we obtain the following exact solution of Eq. (26)

$$y(\xi) = a_0 - 2kab\frac{\exp[a(\xi+\xi_0)]}{1 - b\exp[a(\xi+\xi_0)]}. \tag{30}$$

Then the exact traveling-wave solution to (3 + 1)-dimensional generalized KP equation can be written as

$$u(x,y,z,t) = a_0 - 2kab\frac{e^{a(kx+\alpha y+\beta z-(\frac{k^3a^2\alpha-\beta^2}{k+\alpha})t+\xi_0)}}{1 - be^{a(kx+\alpha y+\beta z-(\frac{k^3a^2\alpha-\beta^2}{k+\alpha})t+\xi_0)}}. \tag{31}$$

When $a_0 = \xi_0 = 0, \quad a = 1, \quad b = -1$, we obtain the exact solution

$$u(x,y,z,t) = 2k\frac{e^{kx+\alpha y+\beta z-(\frac{k^3\alpha-\beta^2}{k+\alpha})t}}{1 + e^{kx+\alpha y+\beta z-(\frac{k^3\alpha-\beta^2}{k+\alpha})t}}. \tag{32}$$

5. Burgers-KP equation

In this section we study the Burgers-KP equation [20]

$$(u_t + uu_x + \mu u_{xx})_x + \lambda u_{yy} = 0. \tag{33}$$

We use the wave transformation

$$u(x,y,t) = y(\xi), \quad \xi = kx + \alpha y - ct, \tag{34}$$

where $k, \; \alpha$ and c are constants, all of them are to be determined.
Substituting (34) into (33), we obtain ordinary differential equation:

$$\mu k^3y_{\xi\xi\xi} + (\lambda\alpha^2 - ck)y_{\xi\xi} + k^2(y_\xi)^2 + k^2yy_{\xi\xi} = 0. \tag{35}$$

When balancing $y_{\xi\xi\xi}$ with $yy_{\xi\xi}$ then gives $l+3 = l+l+2 \Longrightarrow l = 1$. Therefore, we may choose

$$y(\xi) = a_0 + a_1 z(\xi). \tag{36}$$

Substituting (36) along with (6) in Eq. (35) and setting all the coefficients of powers z to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$a_1 = -2\mu k b, \quad c = \frac{\mu k^3 a + k^2 a_0 + \lambda \alpha^2}{k}, \tag{37}$$

where a_0, a, b, k, α and β are arbitrary constants.
Assuming $a > 0$ and choosing $b < 0$. Therefore, using solution (7) of Eq. (6), ansatz (36) , we obtain the following exact solution of Eq. (35)

$$y(\xi) = a_0 - 2\mu k a b \frac{\exp[a(\xi + \xi_0)]}{1 - b\exp[a(\xi+\xi_0)]}. \tag{38}$$

Then the exact traveling-wave solution to Burgers-KP equation can be written as

$$u(x,y,t) = a_0 - 2\mu k a b \frac{e^{a(kx+\alpha y - (\frac{\mu k^3 a + k^2 a_0 + \lambda\alpha^2}{k})t + \xi_0)}}{1 - b e^{a(kx+\alpha y - (\frac{\mu k^3 a + k^2 a_0 + \lambda\alpha^2}{k})t + \xi_0)}}. \tag{39}$$

When $a_0 = \xi_0 = 0, \quad a = 1, \quad b = -1, \quad \alpha = k$, we obtain the exact solution

$$u(x,y,t) = 2\mu k \frac{e^{kx+ky-(\mu k^2+\lambda k)t}}{1 + e^{kx+ky-(\mu k^2+\lambda k)t}}. \tag{40}$$

6. Coupled Higgs field equation

The coupled Higgs field equation [21]

$$u_{tt} - u_{xx} - \alpha u + \beta |u|^2 u - 2uv = 0, \tag{41}$$

$$v_{tt} + v_{xx} - \beta(|u|^2)_{xx} = 0,$$

describes a system of conserved scalar nucleons interacting with a neutral scalar meson. Here, real constant v represents a complex scalar nucleon field and u a real scalar meson field. Eq. (12) is the coupled nonlinear Klein-Gordon equation for $\alpha < 0,\ \beta < 0$ and the coupled Higgs field equation for $\alpha > 0,\ \beta > 0$. The existence of N-soliton solutions for Eq. (12) has been shown by Hirota's bilinear method [22]. To find exact solutions of coupled Higgs field equation (41), first we make the transformation

$$u(x,t) = e^{i\theta} f(\xi), \quad v(x,t) = g(\xi), \tag{42}$$

where $\theta = kx + \omega t,\ \xi = x + ct$, we have a relation $k = \omega c$ and reduce system (41) to the following system of ordinary differential equations

$$(\omega^2(c^2-1)-\alpha)f(\xi) + (c^2-1)f''(\xi) + \beta f^3(\xi) - 2f(\xi)g(\xi) = 0, \tag{43}$$

$$(c^2+1)g''(\xi) - \beta(f^2(\xi))'' = 0. \tag{44}$$

Integrating Eq. (44) twice with respect to ξ, then we have

$$g(\xi) = \frac{R + \beta f^2(\xi)}{c^2+1}, \tag{45}$$

where R is the second integration constant and the first one is taken to zero.
Inserting Eq. (45) into Eq. (43) yields

$$(\omega^2(c^2-1) - \alpha - \frac{2R}{c^2+1})f(\xi) + (c^2-1)f''(\xi) + \beta(1 - \frac{2}{c^2+1})f^3(\xi) = 0. \tag{46}$$

When balancing f'' with f^3 then gives $l + 2 = 3l \Longrightarrow l = 1$. Therefore, we may choose

$$f(\xi) = a_0 + a_1 z(\xi). \tag{47}$$

Substituting (47) into (46) using (6) yields a set of algebraic equations for $a_0,\ a_1,\ \omega,\ a,\ b,\ c,\ R:$

$$2(c^2-1)b^2 a_1 + \beta(1 - \frac{2}{c^2+1})a_1^3 = 0, \tag{48}$$

$$3(c^2-1)aba_1 + 3\beta(1 - \frac{2}{c^2+1})a_0 a_1^2 = 0,$$

$$(c^2-1)a^2 a_1 + (\omega^2(c^2-1) - \alpha - \frac{2R}{c^2+1})a_1 + 3\beta(1 - \frac{2}{c^2+1})a_0^2 a_1 = 0,$$

$$(\omega^2(c^2-1) - \alpha - \frac{2R}{c^2+1})a_0 + \beta(1 - \frac{2}{c^2+1})a_0^3 = 0.$$

With the aid of Maple, we shall find the special solution of the above system

$$R = -\frac{1}{4}(c^2+1)((1-c^2)(2\omega^2-a^2)+2\alpha),\ \ b = \pm\sqrt{-\frac{\beta}{2(c^2+1)}}a_1,\ \ a_0 = \mp a\sqrt{-\frac{c^2+1}{2\beta}}, \tag{49}$$

where a, c, ω and a_1 are arbitrary constants.
Assuming $a > 0$ and choosing $b < 0$. Therefore, using solution (7) of Eq. (6), ansatz (47) , we obtain the following exact solution of Eq. (46)

$$f(\xi) = a\sqrt{2(c^2+1)}(\frac{a_1 \exp(a(\xi+\xi_0))}{\sqrt{2(c^2+1)} - \sqrt{-\beta}a_1 \exp(a(\xi+\xi_0))} \pm \frac{i}{2\sqrt{\beta}}). \tag{50}$$

Then the exact traveling-wave solution to coupled Higgs field equation can be written as

$$u(x,t) = a\sqrt{2(c^2+1)}\left(\frac{a_1 e^{a(x+ct+\xi_0)}}{\sqrt{2(c^2+1)} - \sqrt{-\beta}a_1 e^{a(x+ct+\xi_0)}} \pm \frac{i}{2\sqrt{\beta}}\right)e^{i(\omega cx+\omega t)}. \quad (51)$$

$$v(x,t) = -\frac{1}{4}((1-c^2)(2\omega^2-a^2)+2\alpha)+2a^2\beta\left(\frac{a_1 e^{a(x+ct+\xi_0)}}{\sqrt{2(c^2+1)} - \sqrt{-\beta}a_1 e^{a(x+ct+\xi_0)}} \pm \frac{i}{2\sqrt{\beta}}\right)^2.$$

Substituting (47) along with (9) in Eq. (46) and setting all the coefficients of powers z to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$R = -\frac{1}{4}(c^2+1)((1-c^2)(2\omega^2-A^2)+2\alpha), \ a_0 = \pm A\sqrt{-\frac{c^2+1}{2\beta}}, \ a_1 = \pm\sqrt{-\frac{2(c^2+1)}{\beta}}, \quad (52)$$

where A, c and ω are arbitrary constants.
Assuming $A < 0$. Therefore, using solution (11) of Eq. (9), ansatz (47), we obtain the following exact solution of Eq. (46)

$$f(\xi) = \pm A\sqrt{-\frac{c^2+1}{2\beta}}\tanh\left(\frac{A}{2}(\xi+\xi_0)\right). \quad (53)$$

Then, the exact solution to coupled Higgs field equation can be written as

$$u(x,t) = \pm A\sqrt{-\frac{c^2+1}{2\beta}}\tanh\left(\frac{A}{2}(x+ct+\xi_0)\right)e^{i(\omega cx+\omega t)}, \quad (54)$$

$$v(x,t) = -\frac{1}{4}((1-c^2)(2\omega^2-A^2)+2\alpha) - \frac{A^2}{2}\tanh^2\left(\frac{A}{2}(x+ct+\xi_0)\right).$$

7. Generalized Zakharov System

In the interaction of laser-plasma the system of Zakharov equation plays an important role. This system attracted many scientists wide interest and attention. In this section, we consider the following generalized Zakharov system

$$iu_t + u_{xx} - 2d_1|u|^2u + 2uv = 0, \quad (a) \quad (55)$$

$$\frac{1}{d_2^2}v_{tt} - v_{xx} + \mu(|u|^2)_{xx} = 0. \quad (b)$$

where the real unknown function $v(x,t)$ is the fluctuation in the ion density about its equilibrium value, and the complex unknown function $u(x,t)$ is the slowly varying

envelope of highly oscillatory electron field.
The parameters d_1, d_2 and μ are real numbers, where d_2 is proportional to the electron sound speed. When $d_1 = 0, \mu = 1$, this system is reduced to the Classical Zakharov system of plasma physics.When the sound speed $d_2 \to \infty$, the so-called subsonic limit, the Zakharov system becomes the cubically nonlinear Schrodinger equation.
If we set $d_2 = 1$ and $\mu = 1$, the generalized Zakharov system becomes [23]

$$iu_t + u_{xx} - 2d_1|u|^2u + 2uv = 0, \quad (a) \tag{56}$$

$$v_{tt} - v_{xx} + (|u|^2)_{xx} = 0. \qquad (b)$$

Let us assume the exact solutions of Eq. (56) in the form

$$u(x,t) = e^{i\theta}y(\xi), \quad v(x,t) = V(\xi), \quad \theta = \alpha x + \beta t, \quad \xi = ik(x - 2\alpha t), \tag{57}$$

where α and β are real constants.
Substituting (57) into Eq. (56), we have

$$-(\beta + \alpha^2)y(\xi) + k^2y''(\xi) - 2d_1y^3(\xi) + 2y(\xi)V(\xi) = 0, \qquad (a) \tag{58}$$

$$(1 - 4\alpha^2)V''(\xi) - (y^2(\xi))'' = 0. \quad (b)$$

Integrating Eq. (58)(b) twice with respect to ξ, then we have

$$V(\xi) = \frac{R + y^2(\xi)}{1 - 4\alpha^2}, \tag{59}$$

where R is second integration constant and the first one is taken to zero.
Inserting Eq. (59) into Eq. (58)(a) yields

$$(\frac{2R}{1 - 4\alpha^2} - \beta - \alpha^2)y(\xi) + k^2y''(\xi) + 2(\frac{1}{1 - 4\alpha^2} - d_1)y^3(\xi) = 0. \tag{60}$$

For the solutions of Eq. (60), we make the following ansatz

$$y(\xi) = \sum_{i=0}^{l} a_i z^i, \quad a_l \neq 0. \tag{61}$$

where a_i are all real constants to be determined, l is a positive integer which can be determined by balancing the highest order derivative term with the highest order nonlinear term after substituting ansatz (61) into Eq. (60), where z satisfies Eq. (6).

Balancing $y_{\xi\xi}$ with y^3 in (60) gives $l+2=3l$, so that $l=1$. This suggests the choice of $y(\xi)$ in Eq. (60) as

$$y(\xi)=a_0+a_1 z(\xi). \tag{62}$$

Substituting (62) along with (6) in Eq. (60) and then setting the coefficients of $z^j(j=3,2,1,0)$ to zero in the resultant expression, we obtain a set of algebraic equations involving a_0, a_1, a, b, α and β as $2k^2b^2a_1+2(\frac{1}{1-4\alpha^2}-d_1)a_1^3=0$,

$$3k^2aba_1+6(\frac{1}{1-4\alpha^2}-d_1)a_0a_1^2=0, \tag{63}$$

$$k^2a^2a_1+(\frac{2R}{1-4\alpha^2}-\beta-\alpha^2)a_1+6(\frac{1}{1-4\alpha^2}-d_1)a_0^2a_1=0,$$
$$(\frac{2R}{1-4\alpha^2}-\beta-\alpha^2)a_0+2(\frac{1}{1-4\alpha^2}-d_1)a_0^3=0.$$

Using Maple gives two sets of solutions

$$R=\frac{1}{4}(2\alpha^2+k^2a^2+2\beta)(1-4\alpha^2),\ b=\frac{a_1}{k}\sqrt{\frac{1+d_1(4\alpha^2-1)}{4\alpha^2-1}},\ a_0=\frac{ka}{2}\sqrt{\frac{4\alpha^2-1}{1+d_1(4\alpha^2-1)}}, \tag{64}$$

where k, a_1, a, α and β are arbitrary constants.

$$R=\frac{1}{4}(2\alpha^2+k^2a^2+2\beta)(1-4\alpha^2),\ b=-\frac{a_1}{k}\sqrt{\frac{1+d_1(4\alpha^2-1)}{4\alpha^2-1}},\ a_0=-\frac{ka}{2}\sqrt{\frac{4\alpha^2-1}{1+d_1(4\alpha^2-1)}}, \tag{65}$$

where k, a_1, a, α and β are arbitrary constants.
Assuming $a>0$ and choosing $b<0$ in case (64). Therefore, using solution (7) of Eq. (6), ansatz (62) , we obtain the following traveling-wave solution of Eq. (60)

$$\begin{aligned} y_1(\xi) &= ka\sqrt{4\alpha^2-1}(\frac{1}{2\sqrt{1+d_1(4\alpha^2-1)}} \\ &+ \frac{a_1\exp[a(\xi+\xi_0)]}{k\sqrt{4\alpha^2-1}-a_1\sqrt{1+d_1(4\alpha^2-1)}\exp[a(\xi+\xi_0)]}). \end{aligned} \tag{66}$$

Assuming $a<0$ and choosing $b>0$ in case (65). Therefore, using solution (8) of Eq. (6), ansatz (62) , we obtain the following traveling-wave solution of Eq. (60)

$$\begin{aligned} y_2(\xi) &= -ka\sqrt{4\alpha^2-1}(\frac{1}{2\sqrt{1+d_1(4\alpha^2-1)}} \\ &+ \frac{a_1\exp[a(\xi+\xi_0)]}{k\sqrt{4\alpha^2-1}-a_1\sqrt{1+d_1(4\alpha^2-1)}\exp[a(\xi+\xi_0)]}). \end{aligned} \tag{67}$$

By using (59) and (66), (67) we have

$$\begin{aligned} V_{1,2}(\xi) &= \frac{1}{4}(2\alpha^2+k^2a^2+2\beta)-k^2a^2(\frac{1}{2\sqrt{1+d_1(4\alpha^2-1)}} \\ &+ \frac{a_1\exp[a(\xi+\xi_0)]}{k\sqrt{4\alpha^2-1}-a_1\sqrt{1+d_1(4\alpha^2-1)}\exp[a(\xi+\xi_0)]})^2. \end{aligned} \tag{68}$$

Thus, we obtain the following traveling-wave solutions of generalized Zakharov system (56)

$$\begin{aligned} u(x,t) &= \pm ka\sqrt{4\alpha^2-1}(\frac{1}{2\sqrt{1+d_1(4\alpha^2-1)}} \\ &+ \frac{a_1 exp[a(ik(x-2\alpha t)+\xi_0)]}{k\sqrt{4\alpha^2-1}-a_1\sqrt{1+d_1(4\alpha^2-1)}exp[a(ik(x-2\alpha t)+\xi_0)]})e^{i(\alpha x+\beta t)}, \end{aligned} \tag{69}$$

$$\begin{aligned} v(x,t) &= \frac{1}{4}(2\alpha^2+k^2a^2+2\beta)-k^2a^2(\frac{1}{2\sqrt{1+d_1(4\alpha^2-1)}} \\ &+ \frac{a_1 exp[a(ik(x-2\alpha t)+\xi_0)]}{k\sqrt{4\alpha^2-1}-a_1\sqrt{1+d_1(4\alpha^2-1)}exp[a(ik(x-2\alpha t)+\xi_0)]})^2. \end{aligned}$$

Substituting (62) along with (9) in Eq. (60) and setting all the coefficients of powers z to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$a_1 = \pm k\sqrt{\frac{4\alpha^2-1}{1+d_1(4\alpha^2-1)}}, \quad A = \mp\frac{2a_0}{k}\sqrt{\frac{1+d_1(4\alpha^2-1)}{4\alpha^2-1}}, \tag{70}$$
$$R = \frac{\beta}{2} - 2\beta\alpha^2 + \frac{\alpha^2}{2} - 2\alpha^4 - a_0^2(1+d_1(4\alpha^2-1)),$$

where β, α, a_0 and k are arbitrary constants.
Therefore, using solution (11) of Eq. (9), ansatz (62) , we obtain the following exact solution of Eq. (60)

$$y_{3,4}(\xi) = \pm a_0\tanh(\frac{a_0}{k}\sqrt{\frac{1+d_1(4\alpha^2-1)}{4\alpha^2-1}}(\xi+\xi_0)). \tag{71}$$

By using (59) and (71), we have

$$\begin{aligned} V_{3,4}(\xi) &= \frac{\frac{\beta}{2}-2\beta\alpha^2+\frac{\alpha^2}{2}-2\alpha^4-a_0^2(1+d_1(4\alpha^2-1))}{4\alpha^2-1} \\ &+ \frac{a_0^2}{4\alpha^2-1}\tanh^2(\frac{a_0}{k}\sqrt{\frac{1+d_1(4\alpha^2-1)}{4\alpha^2-1}}(\xi+\xi_0)). \end{aligned} \tag{72}$$

Thus, we obtain the following traveling-wave solutions of generalized Zakharov system (56)

$$u(x,t) = \pm a_0 \tanh\left(\frac{a_0}{k}\sqrt{\frac{1+d_1(4\alpha^2-1)}{4\alpha^2-1}}(ik(x-2\alpha t)+\xi_0)\right)e^{i(\alpha x+\beta t)}, \tag{73}$$

$$\begin{aligned} v(x,t) &= \frac{\frac{\beta}{2}-2\beta\alpha^2+\frac{\alpha^2}{2}-2\alpha^4-a_0^2(1+d_1(4\alpha^2-1))}{4\alpha^2-1} \\ &+ \frac{a_0^2}{4\alpha^2-1}\tanh^2\left(\frac{a_0}{k}\sqrt{\frac{1+d_1(4\alpha^2-1)}{4\alpha^2-1}}(ik(x-2\alpha t)+\xi_0)\right). \end{aligned}$$

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