LINEAR OPERATORS ASSOCIATED WITH A SUBCLASS OF HYPERGEOMETRIC MEROMORPHIC UNIFORMLY CONVEX FUNCTIONS

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Abstract. Making use of certain linear operator, we define a new subclass of meromorphically uniformly convex functions with positive coefficients and obtain coefficient estimates, growth and distortion theorem, extreme points, closure theorems and radii of starlikeness and convexity for the new subclass.

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1. Introduction

Let Σ denote the class of meromorphic functions f normalized by

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n, \tag{1}$$

which are analytic and univalent in the punctured unit disk $U = \{z : 0 < |z| < 1\}$. For $0 \leq \beta$, we denote by $S^*(\beta)$ and $k(\beta)$, the subclasses of Σ consisting of all meromorphic functions which are, respectively, starlike of order β and convex of order β in U (cf. e.g., [1, 2, 4, 12]).

For functions $f_j(z)(j = 1; 2)$ defined by

$$f_j\,(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_{n,j} z^n, \tag{2}$$

we denote the Hadamard product (or convolution) of $f_1(z)$ and $f_2(z)$ by

$$(f_1 * f_2) = \frac{1}{z} + \sum_{n=1}^{\infty} a_{n,1} a_{n,2} z^n. \tag{3}$$

Let us define the function $\tilde{\phi}(a, c; z)$ by

$$\tilde{\phi}(a, c; z) = \frac{1}{z} + \sum_{n=0}^{\infty} \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| a_n z^n, \tag{4}$$

for $c \neq 0, -1, -2, ...,$ and $a \in \mathbf{C}/\{0\}$, where $(\lambda)n = \lambda(\lambda+1)_{n+1}$ is the Pochhammer symbol. We note that

$$\tilde{\phi}(a, c; z) = \frac{1}{z} {}_2F_1(1, a, c; z)$$

where

$${}_2F_1(b, a, c; z) = \sum_{n=0}^{\infty} \frac{(b)_n (a)_n}{(c)_n} \frac{z^n}{n!}$$

is the well-known Gaussian hypergeometric function. Corresponding to the function $\tilde{\phi}(a, c; z)$, using the Hadamard product for $f \in \Sigma$, we define a new linear operator $L^*(a, c)$ on Σ by

$$L^*(a, c) f(z) = \tilde{\phi}(a, c; z) * f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| a_n z^n. \tag{5}$$

The meromorphic functions with the generalised hypergeometric functions were considered recently by Dziok and Srivastava [5], [6], Liu [8], Liu and Srivastava [9], [10], [11], Cho and Kim [3].

For a function $f \in L^*(a, c) f(z)$ we define

$$I^0 (L^*(a, c) f(z)) = L^*(a, c) f(z),$$

and for $k = 1, 2, 3, ...,$

$$\begin{aligned} I^k (L^*(a, c) f(z)) &= z \left(I^{k-1} L^*(a, c) f(z) \right)' + \frac{2}{z} \\ &= \frac{1}{z} + \sum_{n=1}^{\infty} n^k \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| a_n z^n. \end{aligned} \tag{6}$$

We note that $I^k\left(L^*\left(a,a\right)f\left(z\right)\right)$ studied by Frasin and Darus [7].

Also, it follows from (6) that

$$z\left(L(a,c)f(z)\right)' = aL(a+1,c)f(z) - \left(a+1\right)L(a,c)f(z).$$

Now, for $\alpha(-1 \le \alpha < 1)$ and $\beta(\beta \ge 1)$, we let $\Sigma^*(\alpha,\beta,k)$ be the subclass of A consisting of the form (1) and satisfying the analytic criterion

$$\Re\left\{\frac{I^{k+1}L^*\left(a,c\right)f\left(z\right)}{I^kL^*\left(a,c\right)f\left(z\right)} - \alpha\right\} > \beta\left|\frac{I^{k+1}L^*\left(a,c\right)f\left(z\right)}{I^kL^*\left(a,c\right)f\left(z\right)} - 1\right|, \quad z \in U \tag{7}$$

where $L^*\left(a,c\right)f\left(z\right)$ is given by (5).

The main objective of this paper is to obtain necessary and sufficient conditions for the functions $f \in \Sigma^*(\alpha,\beta,k)$. Furthermore, we obtain extreme points, growth and distortion bounds and closure properties for the class $\Sigma^*(\alpha,\beta,k)$.

2.Basic properties

In this section we obtain necessary and sufficient conditions for functions f in the class $\Sigma^*(\alpha,\beta,k)$.

Theorem 1. *A function f of the form (1) is in $\Sigma^*(\alpha,\beta,k)$ if*

$$\sum_{n=1}^{\infty} n^k\left[n\left(1+\beta\right) - \left(\beta+\alpha\right)\right]\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}\left|a_n\right| \le 1-\alpha \tag{8}$$

$-1 \le \alpha < 1$ *and* $\beta \ge 1$.

Proof. It suffices to show that

$$\beta\left|\frac{I^{k+1}L^*\left(a,c\right)f\left(z\right)}{I^kL^*\left(a,c\right)f\left(z\right)} - 1\right| - \Re\left\{\frac{I^{k+1}L^*\left(a,c\right)f\left(z\right)}{I^kL^*\left(a,c\right)f\left(z\right)} - 1\right\} \le 1-\alpha.$$

We have

$$\beta\left|\frac{I^{k+1}L^*(a,c)f(z)}{I^kL^*(a,c)f(z)}-1\right|-\Re\left\{\frac{I^{k+1}L^*(a,c)f(z)}{I^kL^*(a,c)f(z)}-1\right\}$$

$$\leq(1+\beta)\left|\frac{I^{k+1}L^*(a,c)f(z)}{I^kL^*(a,c)f(z)}-1\right|\leq\frac{(1+\beta)\sum\limits_{n=1}^{\infty}n^k(n-1)\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}\left|a_n\right|\left|z\right|^n}{\frac{1}{|z|}-\sum\limits_{n=1}^{\infty}n^k\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n\left|z\right|^n}$$

Letting $z\to 1$ along the real axis, we obtain

$$\leq\frac{(1+\beta)\sum\limits_{n=1}^{\infty}n^k(n-1)\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}\left|a_n\right|}{1-\sum\limits_{n=1}^{\infty}n^k\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}\left|a_n\right|}$$

This last expression is bounded above by $(1-\alpha)$ if

$$\sum_{n=1}^{\infty}n^k\left[n(1+\beta)-(\beta+\alpha)\right]\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}\left|a_n\right|\leq 1-\alpha$$

Hence the theorem.

Our assertion in Theorem 1 is sharp for functions of the form

$$f_n(z)=\frac{1}{z}+\sum_{n=1}^{\infty}\frac{(1-\alpha)\left|(c)_{n+1}\right|}{n^k\left[n(1+\beta)-(\beta+\alpha)\right]\left|(a)_{n+1}\right|}z^n, \tag{9}$$

$(n\geq 1; k\in N_0)$.

Corollary 1. *Let the functions f be defined by (5) and let $f\in A$, then*

$$a_n\leq\frac{(1-\alpha)(c)_{n+1}}{n^k\left[n(1+\beta)-(\beta+\alpha)\right](a)_{n+1}}, \tag{10}$$

$(n\geq 1; k\in N_0,)$.

Theorem 2. *Let f define by (1) and $g(z) = \frac{1}{z} + \sum\limits_{n=1}^{\infty} b_n z^n$ be in the class $\Sigma^*(\alpha, \beta, k)$. Then the function h defined by*

$$h(z) = (1-\lambda) f(z) + \lambda g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} q_n z^n, \tag{11}$$

where $q_n = (1-\lambda) a_n + \lambda b_n$, $0 \leq \lambda < 1$ is also in the class $\Sigma^*(\alpha, \beta, k)$.

3.Growth and Distortion Theorem

Theorem 3. *Let the function f defined by (6) be in the class $\Sigma^*(\alpha, \beta, k)$. Then*

$$\frac{1}{r} - r \leq |f(z)| \leq \frac{1}{r} + r \tag{12}$$

Equality holds for the function

$$f(z) = \frac{1}{z} + z.$$

Proof. Since $f \in S^*(\alpha, \beta, k)$, by Theorem 1,

$$\sum_{n=1}^{\infty} n^k \left[n(1+\beta) - (\beta+\alpha)\right] \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} |a_n| \leq 1-\alpha$$

Now

$$(1-\alpha) \sum_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n = \sum_{n=1}^{\infty} (1-\alpha) \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n \leq$$

$$\sum_{n=1}^{\infty} n^k \left[n(1+\beta) - (\beta+\alpha)\right] \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} |a_n| \leq 1-\alpha$$

and therefore

$$\sum_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n \leq 1.$$

Since $f(z) = \frac{1}{z} + \sum\limits_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n z^n$,

$$|f(z)| = \left| \frac{1}{z} + \sum_{n=1}^{\infty} \frac{(a)_{n+1}}{(c)_{n+1}} a_n z^n \right| \leq \frac{1}{|z|} + \sum_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n |z|^n$$

$$\leq \frac{1}{r} + r \sum_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n \leq \frac{1}{r} + r$$

and

$$|f(z)| = \left| \frac{1}{z} - \sum_{n=1}^{\infty} \frac{(a)_{n+1}}{(c)_{n+1}} a_n z^n \right| \geq \frac{1}{|z|} - \sum_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n |z|^n$$

$$\geq \frac{1}{r} - r \sum_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n \geq \frac{1}{r} - r$$

which yields the theorem.

Theorem 4. *Let the function f defined by (6) be in the class $\Sigma^*(\alpha, \beta, k)$. Then*

$$\frac{1}{r^2} - 1 \leq |f'(z)| \leq \frac{1}{r^2} + 1, \tag{13}$$

Equality holds for the function $f(z) = \frac{1}{z} + z$.

Proof. we have

$$|f'(z)| = \left| \frac{-1}{z^2} + \sum_{n=1}^{\infty} n \frac{(a)_{n+1}}{(c)_{n+1}} a_n z^{n-1} \right| \leq \frac{1}{|z|^2} - \sum_{n=1}^{\infty} n \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n |z|^{n-1}$$

$$\leq \frac{1}{r^2} - \sum_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} n a_n \tag{14}$$

Since $f(z) \in \Sigma^*(\alpha, \beta, k)$, we have

$$(1-\alpha)\sum_{n=1}^{\infty}\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}na_n \leq \sum_{n=1}^{\infty}n^{k-1}\left[n\left(1+\beta\right)-\left(\beta+\alpha\right)\right]\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}na_n \leq 1-\alpha$$

Hence,

$$\sum_{n=1}^{\infty}\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}na_n \leq 1. \tag{15}$$

Substituting (14) in (15), we get

$$|f'(z)| \leq \frac{1}{r^2}+1$$

$$|f'(z)| \geq \frac{1}{r^2}-1$$

This completes the proof.

4.Radii Of Starlikeness and Convexity

The radii of Starlikeness and convexity for the class for the class $\Sigma^*(\alpha, \beta, k)$ is given by the following theorems.

Theorem 5. *If the function f be defined by (6) is in the class $\Sigma^*(\alpha, \beta, k)$ then f is meromorphically starlike of order $\delta(0 \leq \delta < 1)$ in $|z-p| < |z| < r_1$, where*

$$r_1 = r_1\left(\alpha, \beta, k\right) = \inf_{n\geq 1}\left\{\frac{n^k\left(1-\delta\right)\left[n\left(1+\beta\right)-\left(\beta+\alpha\right)\right]}{\left(n+2-\delta\right)\left(1-\alpha\right)}\right\}^{\frac{1}{n+1}}. \tag{16}$$

The result is sharp for the function f given by (9).

Proof. It suffices to prove that

$$\left|\frac{z\left(I^k f\left(z\right)\right)'}{I^k f\left(z\right)}+1\right| < 1-\delta \tag{17}$$

for $|z| < r_1$. The left hand side we have

$$\left|\frac{z\left(I^k f(z)\right)'}{I^k f(z)} + 1\right| = \left|\frac{\sum\limits_{n=1}^{\infty} n^k (n+1) \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n z^n}{\frac{1}{z} + \sum\limits_{n=1}^{\infty} n^k \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n z^n}\right| \leq \frac{\sum\limits_{n=1}^{\infty} n^k (n+1) \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n |z|^n}{\frac{1}{|z|} - \sum\limits_{n=1}^{\infty} n^k \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n |z|^n} \tag{18}$$

The last expression is less than $1-\delta$ if

$$\sum_{n=1}^{\infty} n^k (n+1) \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n |z|^n \leq (1-\delta)\left(\frac{1}{|z|} - \sum_{n=1}^{\infty} n^k \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n |z|^n\right) \tag{19}$$

or

$$\sum_{n=1}^{\infty} n^k \frac{(n+2-\delta)}{(1-\delta)} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} |z|^{n+1} \leq 1 \tag{20}$$

with the aid of (8) and (20) is true if

$$\sum_{n=1}^{\infty} n^k \frac{(n+2-\delta)}{(1-\delta)} |z|^{n+1} \leq \frac{n^k [n(1+\beta) - (\beta+\alpha)]}{(1-\alpha)} \tag{21}$$

$n \geq 1$.
Solving (21) for $|z|$, we obtain

$$|z| < \left\{\frac{n^k (1-\delta)[n(1+\beta) - (\beta+\alpha)]}{(n+2-\delta)(1-\alpha)} \frac{(a)_{n+1}}{(c)_{n+1}}\right\}^{\frac{1}{n+1}}.$$

This completes the proof of Theorem 5.

Theorem 6. *If the function f be defined by (6) is in the class $\Sigma^*(\alpha, \beta, k)$ then $f(z)$ is meromorphically convex of order $\delta (0 \leq \delta < 1)$ in $|z| < r_2$, where*

$$r_2 = r_2(\alpha, \beta, k) =$$

$$\inf_{n \geq 1} \left\{\frac{\mathrm{n}^{\mathrm{k}-1} (1-\delta)[n(1+\beta) - (\beta+\alpha)]}{(n+2-\delta)(1-\alpha)}\right\}^{\frac{1}{n+1}}. \tag{22}$$

The result is sharp for the function f given by (9).

Proof. By using the technique employed in the proof of Theorem , we can show that

$$\left|\frac{zf''(z)}{f'(z)}+2\right| \leq (1-\delta) \tag{23}$$

for $|z| < r_2$, with the aid of Theorem 1. Thus we have the assertion of Theorem 6.

5.Convex Linear Combinations

Our next result involves linear combinations of several functions of the type (9).

Theorem 7. *Let*

$$f_0\left(z\right) = \frac{1}{z} \tag{24}$$

and

$$f_n\left(z\right) = \frac{1}{z} + \frac{(1-\alpha)\left|(c)_{n+1}\right|}{n^k\left[n\left(1+\beta\right)-(\beta+\alpha)\right]\left|(a)_{n+1}\right|}z^n, \tag{25}$$

$n \geq 1, -1 < \alpha \leq 1$, $\beta \geq 0$ *and* $k \geq 0$.
Then $f(z) \in S^*(\alpha, \beta, k)$ *if and only if it can be expressed in the form*

$$f(z) = \sum_{n=0}^{\infty} \lambda_n f_n(z) \tag{26}$$

where $\lambda_n \geq 0$ and $\sum\limits_{n=0}^{\infty} \lambda_n = 1$.

Proof. From (24), (25) and (26), it is easily seen that

$$f(z) = \sum_{n=0}^{\infty} \lambda_n f_n(z) = \frac{\lambda_0}{z} + \sum_{n=1}^{\infty} \frac{\lambda_n\left(1-\alpha\right)\left|(c)_{n+1}\right|}{n^k\left[n\left(1+\beta\right)-(\beta+\alpha)\right]\left|(a)_{n+1}\right|}z^n, \tag{27}$$

Since

$$\sum_{n=1}^{\infty} \frac{n^k \left[n\left(1+\beta\right)-\left(\beta+\alpha\right)\right] \left|(a)_{n+1}\right|}{\left(1-\alpha\right)\left|(c)_{n+1}\right|} \cdot \frac{\lambda_n \left(1-\alpha\right)\left|(c)_{n+1}\right|}{n^k \left[n\left(1+\beta\right)-\left(\beta+\alpha\right)\right] \left|(a)_{n+1}\right|}$$

$$= \sum_{n=1}^{\infty} \lambda_n = 1 - \lambda_0 \leq 1.$$

It follows from Theorem 1 that $f \in S^*(\alpha, \beta, k)$.

Conversely, let us suppose that $f \in S^*(\alpha, \beta, k)$. Since

$$a_n \leq \frac{\left(1-\alpha\right)\left|(c)_{n+1}\right|}{n^k \left[n\left(1+\beta\right)-\left(\beta+\alpha\right)\right] \left|(a)_{n+1}\right|},$$

$n \geq 1, -1 < \alpha \leq 1$, $\beta \geq 0$ and $k \geq 0$.
Setting $\lambda_n = \frac{n^k[n(1+\beta)-(\beta+\alpha)]\left|(a)_{n+1}\right|}{(1-\alpha)\left|(c)_{n+1}\right|}$, $n \geq 1$, $k \geq 0$ and $\lambda_0 = 1 - \sum\limits_{n=1}^{\infty} \lambda_n$.
It follows that $f(z) = \sum\limits_{n=0}^{\infty} \lambda_n f_n(z)$. This completes the proof of the theorem.

Finally, we prove the following:

Theorem 8. *The class $S^*(\alpha, \beta, k)$ is closed under convex linear combinations.*

Proof. Suppose that the function $f_1(z)$ and $f_2(z)$ defined by

$$f_j\left(z\right) = \frac{1}{z} - \sum_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_{n,j} z^n, \qquad \left(j = 1, 2;\ z \in U\right). \tag{28}$$

are in the class $S^*(\alpha, \beta, k)$. Setting

$$f\left(z\right) = \mu f_1\left(z\right) + \left(1-\mu\right) f_2\left(z\right), \qquad \left(0 \leq \mu < 1\right) \tag{29}$$

we find from (28) that

$$f\left(z\right) = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} \left\{\mu a_{n,1} + \left(1-\mu\right) a_{n,2}\right\} z^n, \tag{30}$$

$(0 \leq \mu < 1,\ z \in U)$.
In view of Theorem 1, we have

$$\sum_{n=0}^{\infty} n^k \left[n\left(1+\beta\right)-\left(\beta+\alpha\right)\right] \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} \left\{\mu a_{n,1}+\left(1-\mu\right) a_{n,2}\right\}$$

$$=\mu \sum_{n=1}^{\infty} n^k \left[n\left(1+\beta\right)-\left(\beta+\alpha\right)\right] \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_{n,1}$$

$$+\left(1-\mu\right) \sum_{n=1}^{\infty} n^k \left[n\left(1+\beta\right)-\left(\beta+\alpha\right)\right] \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_{n,2}$$

$$\leq \mu\left(1-\alpha\right)+\left(1-\mu\right)\left(1-\alpha\right)=\left(1-\alpha\right).$$

which shows that $f(z) \in S^{*}(\alpha,\beta,k)$. Hence the theorem.

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