AMENABILITY AND WEAK AMENABILITY OF LIPSCHITZ OPERATORS ALGEBRAS

A.A. Shokri, A. Ebadian and A.R. Medghalchi

Abstract. In a recent paper by H.X. Cao, J.H. Zhang and Z.B. Xu a α-Lipschitz operator from a compact metric space into a Banach space A is defined and characterized in a natural way in the sence that $F : K \to A$ is a α-Lipschitz operator if and only if for each $\sigma \in X^*$ the mapping $\sigma o F$ is a α-Lipschitz function. The Lipschitz operators algebras $L^\alpha(K, A)$ and $l^\alpha(K, A)$ are developed here further, and we study their amenability and weak amenability of these algebras. Moreover, we prove an interesting result that $L^\alpha(K, A)$ and $l^\alpha(K, A)$ are isometrically isomorphic to $L^\alpha(K)\check{\otimes}A$ and $l^\alpha(K)\check{\otimes}A$ respectively.

2000 *Mathematics Subject Classification*: 47B48, 46J10.

1. Introduction

Let (K, d) be compact metric space with at least two elements and $(X, \| . \|)$ be a Banach space over the scaler field $\mathbf{F}$ (= R or C). For a constant $\alpha > 0$ and an operator $T : K \to X$, set

$$L_\alpha(T) := \sup_{s\neq t} \frac{\| T(t) - T(s) \|}{d(s, t)^\alpha}, \tag{1}$$

which is called the Lipschitz constant of T. Define

$$\begin{aligned} T_\alpha(x, y) &= \frac{T(x) - T(y)}{d(x, y)^\alpha} \quad , \quad x \neq y \\ L^\alpha(K, X) &= \{T : K \to X \quad : \quad L_\alpha(T) < \infty\} \end{aligned}$$

and

$$l^\alpha(K, X) = \{T : K \to X \quad : \quad \| T_\alpha(x, y) \| \to 0 \quad as \quad d(x, y) \to 0\}.$$

The elements of $L^\alpha(K,X)$ and $l^\alpha(K,X)$ are called big and little Lipschitz operators, respectively [1].

Let $C(K,X)$ be the set of all continuous operators from K into X and for each $T \in C(K,X)$, define

$$\| T \|_\infty = \sup_{x\in K} \| T(x) \|$$

For S, T in $C(K,X)$ and λ in F, define

$$(S+T)(x) = S(x) + T(x) \quad , \quad (\lambda T)(x) = \lambda T(x), \quad (x \in X).$$

It is easy to see that $(C(K,X), \| \,.\, \|_\infty)$ becomes a Banach space over F and $L^\alpha(K,X)$ is a linear subspace of $C(K,X)$. For each element T of $L^\alpha(K,X)$, define $\| T \|_\alpha = L_\alpha(T) + \| T \|_\infty$.

In their papers[3,4], Cao, Zhang and Xu proved that $(L^\alpha(K,X), \| \,.\, \|_\alpha)$ is a Banach space over F and $l^\alpha(K,X)$ is a closed linear subspace of $(L^\alpha(K,X), \| \,.\, \|_\alpha)$. Now, let $(A, \| \,.\, \|)$ be a unital Banach algebra with unit e. In this paper, we show that $(L^\alpha(K,A), \| \,.\, \|_\alpha)$ is a Banach algebra under pointwise and scalar multiplication and $l^\alpha(K,A)$ is a closed linear subalgebra of $(L^\alpha(K,A), \| \,.\, \|_\alpha)$ and study many aspects of these algebras. The spaces $L^\alpha(K,A)$ and $l^\alpha(K,A)$ are called big and little Lipschitz operators algebras. Note that Lipschitz operators algebras are, in fact, extensions of Lipschitz algebras. Sherbert [12, 13], Weaver [14, 15], Honary and Mahyar [7], Johnson [9], Alimohammadi and Ebadian[1], Ebadian[6], Bade, Curtis and Dales[2], studied some properties of Lipschitz algebras. Finally, we will study (weak) amenability of Lipschitz operators algebras.

2.Characterizations Of Lipschitz Operators Algebras

In this section, let (K,d) be a compact metric space which has at least two elements and $(A, \| \,.\, \|)$ to denote a unital Banach algebra over the scalar field F(= R or C).

Theorem 2.1. *$L^\alpha(K,A)$, $\| \,.\, \|_\alpha$) is a Banach algebra over* F *and $l^\alpha(K,A)$ is a closed linear subspace of ($L^\alpha(K,A)$, $\| \,.\, \|_\alpha$).*

Proof. As we have already $L^\alpha(K,A)$ is a Banach space and $l^\alpha(K,A)$ is a closed linear subspace if it. Now let T, $S \in L^\alpha(K,A)$, and define

$$(TS)(t) = T(t)S(t) \quad (t \in K).$$

Then

$$\begin{aligned}\| TS \|_\alpha &= \| TS \|_\infty + L_\alpha(TS) \\ &\leq \| T \|_\infty \| S \|_\infty + \sup_{t\neq s} \frac{\| T(t)S(t) - T(s)S(s) \|}{d(t,s)^\alpha} \\ &\leq \| T \|_\infty \| S \|_\infty + \| T \|_\infty L_\alpha(S) + \| S \|_\infty L_\alpha(T) \\ &\leq (\| T \|_\infty + L_\alpha(T))(\| S \|_\infty + L_\alpha(S)) \\ &= \| T \|_\alpha \| S \|_\alpha .\end{aligned}$$

So that we see that $(L^\alpha(K,A), \| \,.\, \|_\alpha)$ is a Banach algebra and $l^\alpha(K,A)$ is a closed linear subspace of $(L^\alpha(K,A), \| \,.\, \|_\alpha)$. △

Theorem 2.2. *Let (K,d) be a compact metric space. Then $L^\alpha(K,A)$ is uniformly dense in $C(K,A)$.*
Proof. Let $f \in C(K,A)$. Then for every $\sigma \in A^*$ we have $\sigma o f \in C(K)$, so that there is $g \in L^\alpha(K)$ such that $\| g - \sigma o f \|_\infty < \varepsilon$. We define, the map $\eta : \mathrm{C} \to A$ by $\eta(\lambda) = \lambda.e$. It is easy to see that $\eta o g \in L^\alpha(K,A)$, and for every $\sigma \in A^*$, we have

$$| \sigma(g(x).e - f(x)) | = | g(x) - (\sigma o f)(x) | < \varepsilon \quad , \quad (x \in K).$$

Therefor $| \sigma(\eta o g - f)(x) | < \varepsilon$ for every $\sigma \in A^*$ and $x \in K$. This implies that $\| (\eta o g - f)(x) \| < \varepsilon$ for every $x \in K$. Therefor, $\| \eta o g - f \|_\infty < \varepsilon$ and the proof is complete. △

Remark 2.3. Let A, B be unital Banach algebras over F. Then the injective tensor $A \check{\otimes} B$ is a unital Banach algebra under norm $\|.\|_\epsilon$ [11].

Theorem 2.4. $L^\alpha(K,A) = \{F : K \to A | \quad \sigma o F \in L^\alpha(K,\mathrm{C}), (\forall \sigma \in A^*)\}$
Proof. Use the principle of Uniform Boundedness. △

For every Banach algebra B, let Φ_B be the space of maximal ideal of B.
Theorem 2.5. *$\Phi_{L^\alpha(K,A)}$ and $\Phi_{l^\alpha(K,A)}$ are identified with K.*
Proof. Similarly to the proof of Lipschitz algebras. △

Theorem 2.6. *Let (K,d) be a compact metric space and A be a unital Banach algebra. Then $L^\alpha(K,A)$ is isometrically isomorphic to $L^\alpha(K) \check{\otimes} A$.*
Proof. It is straightforward to prove that the mapping $V : L^\alpha(K) \times A \to$

$L^\alpha(K, A)$ defined by

$$\begin{aligned} V(f,a) &= fa \quad (f \in L^\alpha(K), \quad a \in A), \\ (fa)(x) &:= f(x)a \quad (x \in K), \end{aligned}$$

is bilinear. Therefor there exists a unique linear map $T : L^\alpha(K)\check{\otimes}A \to L^\alpha(K, A)$ such that $T(f \otimes a) = V(f, a)$, [11]. We have

$$\begin{aligned} \| T(f \otimes a) \|_\alpha &= \| V(f,a) \|_\alpha = \| fa \|_\alpha = \| fa \|_\infty + L_\alpha(fa) \\ &= \| f \|_\infty \| a \| + L_\alpha(f) \| a \| \\ &= \| f \|_\alpha \| a \| = \| f \otimes a \|_\varepsilon . \end{aligned}$$

Therefor T is a linear isometry of $L^\alpha(K)\check{\otimes}A$ into $L^\alpha(K, A)$. Now, we show that the range of T, R_T is a closed and dense subset of $L^\alpha(K, A)$. It is easy to see that R_T is closed. Let $f \in L^\alpha(K, A)$ and $\gamma > 0$. There exist $a_1, \ldots, a_n \in A$ such that $X := f(K) \subset \bigcup_{i=1}^n B(a_i, \gamma)$. Set $U_j = f^{-1}(B(a_j, \gamma))$ where $j = 1, \cdots, n$. Then there exist $f_1, \ldots, f_n \in L^\alpha(K, A)$ and $\sigma \in A^*$such that $supp(f_j) \subset U_j$ for $j = 1, \ldots, n$ and $\sigma o(f_1 + \ldots + f_n) = 1$. For every $x \in K$ we have,

$$\begin{aligned} & \| f(x) - ((\sigma o f_1)a_1 + \cdots + (\sigma o f_n)a_n)(x) \| \\ = & \| f(x)\Big((\sigma o f_1)(x) + \cdots + (\sigma o f_n)(x)\Big) - \Big((\sigma o f_1)(x)a_1 + \cdots + (\sigma o f_n)(x)a_n\Big) \| \\ = & \| (\sigma o f_1)(x)(f(x) - a_1) + \cdots + (\sigma o f_n)(x)(f(x) - a_n) \| \\ \leq & \sum_{i=1}^n | (\sigma o f_i)(x) | \| f(x) - a_i \| < \gamma, \end{aligned}$$

since $suppf_j \subset U_j$. Therefore,

$$\| f - ((\sigma o f_1)a_1 + \cdots + (\sigma o f_n)a_n) \|_\alpha < \gamma.$$

This implies that

$$\| f - \sum_{i=1}^n T(\sigma o f_i, a_i) \|_\alpha < \gamma.$$

We conclude that $\bar{R}_T = L^\alpha(K, \alpha)$. Let τ and τ' be topologies on $L^\alpha(K)\check{\otimes}A$ and $L^\alpha(K, A)$ respectively. Let $U \in \tau$, we show that $T(U) \in \tau'$. Let p be a limit point in $L^\alpha(K, A) \backslash T(U)$. Then there exists a sequence $\{p_n\}$ in $L^\alpha(K, A) \backslash T(U)$ converges to p. Since T is onto, there is a sequence $\{q_n\}$ in $L^\alpha(K)\check{\otimes}A$ such that $T(q_n) = p_n$. Therefore $T(q_n)$ converges to p in $L^\alpha(K)$. Since $q_n \in L^\alpha(K)\check{\otimes}A$, we can find $m \in \mathrm{N}$, $f_j^{(n)} \in L^\alpha(K)$ and $a_j^{(n)} \in A$ such that whenever $1 \leq j \leq m$ we have

$$T(q_n) = \sum_{j=1}^{m} f_j^{(n)} a_j^{(n)}. \tag{1}$$

Also, since $q \in L^\alpha(K)\check{\otimes}A$ there exist $r \in \mathrm{N}$, $g_i \in L^\alpha(K)$ and $b_i \in A$ such that

$$p = T(q) = \sum_{i=1}^{r} g_i b_i. \tag{2}$$

Since $\|T(q_n) - p\|_\alpha \to 0$ as $n \to \infty$, for every positive number γ there exists a positive integer N such that

$$\|\sum_{j=1}^{m} f_j^{(n)} a_j^{(n)} - \sum_{i=1}^{r} g_i b_i\|_\alpha < \gamma, \tag{3}$$

when $n \geq N$. By applying (3), we have

$$\begin{aligned} & \sup_{(x\in K)} \|\sum_{j=1}^{m} f_j^{(n)}(x) a_j^{(n)} - \sum_{i=1}^{r} g_i(x) b_i\| \\ + & \sup_{(x\neq y)} \frac{1}{d(x,y)^\alpha} \|\sum_{j=1}^{m} f_j^{(n)}(x) a_j^{(n)} - \sum_{i=1}^{r} g_i(x) b_i - \sum_{j=1}^{m} f_j^{(n)}(y) a_j^{(n)} + \sum_{i=1}^{r} g_i(y) b_i\| \\ < & \gamma. \end{aligned}$$

Therefore if $\sigma \in A^*$ with $\|\sigma\| \leq 1$ then

$$\begin{aligned} & \sup_{(x\in K)} \|\sum_{j=1}^{m} f_j^{(n)}(x)\sigma(a_j^{(n)}) - \sum_{i=1}^{r} g_i(x)\sigma(b_i)\| \\ + & \sup_{(x\neq y)} \frac{1}{d(x,y)^\alpha} \|\sum_{j=1}^{m} f_j^{(n)}(x)\sigma(a_j^{(n)}) - \sum_{i=1}^{r} g_i(x)\sigma(b_i) \sum_{j=1}^{m} f_j^{(n)}(y)\sigma(a_j^{(n)}) \\ + & \sum_{i=1}^{r} g_i(y)\sigma(b_i)\| \\ < & \gamma. \end{aligned}$$

This implies that

$$\|\sum_{j=1}^{m} f_j^{(n)} \sigma(a_j^{(n)}) - \sum_{i=1}^{r} g_i \sigma(b_i)\|_\alpha < \gamma \tag{4}$$

Now by using (4), for every $\phi \in L^\alpha(K)^*$ with $\|\phi\|_\alpha \leq 1$ we have,

$$|\phi(\sum_{j=1}^{m} f_j^{(n)}\sigma(a_j^{(n)}) - \sum_{i=1}^{r} g_i\sigma(b_i))| < \gamma,$$

hence

$$|\sum_{j=1}^{m} \phi(f_j^{(n)})\sigma(a_j^{(n)}) - \sum_{i=1}^{r} \phi(g_i)\sigma(b_i)| < \gamma. \qquad (5)$$

By (5), we conclude

$$\sup|\sum_{j=1}^{m} \phi(f_j^{(n)})\sigma(a_j^{(n)}) - \sum_{i=1}^{r} \phi(g_i)\sigma(b_i)| < \gamma, \quad \|\sigma\| \leq 1, \quad \|\phi\|_\alpha \leq 1. \qquad (6)$$

Therefore $\|q_n - q\|_\epsilon \leq \gamma$ and hence $q_n \to q$ or $q_n \to T^{(-1)}(p)$ in $L^\alpha(K)\check{\otimes}A$. This show that $p \in T(U)^c$ and the proof is complete. $\triangle$

Remark 2.7. By using the above theorem we can prove that $l^\alpha(K, A) \cong l^\alpha(K)\check{\otimes}A$.

3.(Weak) Amenability Of $L^\alpha(K, A)$

Let A be a Banach algebra and X be a Banach A-module over F. The linear map $D : A \to X$ is called an X-derivation on A, if $D(ab) = D(a).b + a.D(b)$, for every $a, b \in A$. The set of all continues X-derivations on A is a vector space over F which is denoted by $Z^1(A, X)$. For each $x \in X$ the map $\delta_x : A \to X$, defined by $\delta_x(a) = a.x - x.a$, is a continues X-derivation on A. The X-derivation $D : A \to X$ is called an inner derivation on A if there exists an $x \in X$ such that $D = \delta_x$. The set of all inner X-derivations on A is a linear subspace of $Z^1(A, X)$ which is denoted by $B^1(A, X)$. The quotient space $Z^1(A, X)/B^1(A, X)$ is denoted by $H^1(A, X)$ and is called the first cohomology group of A with coefficients in X.

Definition 3.1. *The Banach algebra A over* F *is called amenable if for every Banach A-module X over* F*, $H^1(A, X^*) = \{0\}$. The Banach algebra A over* F *is called weakly amenable if $H^1(A, A^*) = \{0\}$.*

The notion of amenability of Banach algebras were first introduced by B. E. Johnson in 1972 [8]. Bade, Curtis and Dales [2], studied the (weak) amenability of Lipschitz algebras in 1987 [2]. In this section, we study the (weak) amenability of $L^{\alpha}(K, A)$.

Definition 3.2. *Let A be a commutative Banach algebra and let $\phi \in \Phi_A \cup \{0\}$. The non-zero linear functional D on A is called point derivation at ϕ if*

$$D(ab) = \phi(a)D(b) + \phi(b)D(a), \quad (a, b \in A).$$

Lemma 3.3. *For each non-isolated point $x \in K$ and $\sigma \in A^*$, if the map $\phi : L^{\alpha}(K, A) \to \mathrm{C}$ is given by*

$$\phi(f) = (\sigma o f)(x), \quad (f \in L^{\alpha}(K, A))$$

then $\phi \in \Phi_{L^{\alpha}(K,A)}$.
*Proof.*Obvious. △

Let (K, d) be a fixed non-empty compact metric space, set

$$\Delta = \{(x, y) \in K \times K : x = y\}, \quad W = K \times K - \Delta.$$

We now examine the amenability and weak amenability of Lipschitz operators algebras $L^{\alpha}(K, A)$ and $l^{\alpha}(K, A)$.

Theorem 3.4. *Let (K, d) be an infinite compact metric space and take $\alpha \in (0, 1]$. Then $L^{\alpha}(K, A)$ is not weakly amenable.*
Proof. Let x be a non-isolated point in K. We define

$$W_x := \{\{(x_n, y_n)\}_{n=1}^{\infty} : (x_n, y_n) \in W, \quad (x_n, y_n) \to (x, x) \quad as \quad n \to \infty\}$$

For the net $w = \{(x_n, y_n)\}_{n=1}^{\infty}$ in W_x and $\sigma \in A^*$, we put

$$\overline{w}(f) = \frac{(\sigma o f)(x_n) - (\sigma o f)(y_n)}{d(x_n, y_n)^{\alpha}}, \quad (f \in L^{\alpha}(K, A))$$

then $\| \overline{w}(f) \|_{\infty} \leq \| \sigma \| \| f \|_{\alpha}$. Hence, $\overline{w}$ is continues. Now set

$$D_w(f) = LIM(\overline{w}(f)) \quad , \quad (f \in L^{\alpha}(K, A))),$$

where $LIM(.)$ is Banach limit [12]. We show that the linear map D_w is a non-zero point derivation at ϕ, which ϕ is given by Lemma 6. We have,

$$\begin{aligned}
D_w(fg) &= LIM(\overline{w}(fg)) \\
&= LIM\frac{(\sigma ofg)(x_n) - (\sigma ofg)(y_n)}{d(x_n, y_n)^\alpha} \\
&= LIM\frac{1}{d(x_n, y_n)^\alpha}\Big[\sigma o\Big(f(x_n)g(x_n) - f(x_n)g(y_n)\Big)\Big] \\
&= LIM\frac{1}{d(x_n, y_n)^\alpha}\Big[\sigma o\Big(f(x_n)(g(x_n) - g(y_n)) \\
&+ g(y_n)(f(x_n) - f(y_n))\Big)\Big] \\
&= (\sigma of)(x)LIM(\overline{w}(g)) + (\sigma og)(x)LIM(\overline{w}(g)) \\
&= \phi(f)D_w(g) + \phi(g)D_w(f)
\end{aligned}$$

Therefor, by the continuity f, g and properties of Banach limit we conclude D_w is a non-zero, continues point derivation at ϕ on $L^\alpha(K, A)$, an so by [5], $L^\alpha(K, A)$ is not weakly amenable. △

Corollary 3.5. *$L^\alpha(K, A)$ is not amenable.*

Definition 3.6. *A subset E of an abelian group G is said to be independent if E has the foolowing property: for every choice of distinct points $x_1, \ldots, x_k$ of E and integers $n_1, \ldots, n_k$, either*

$$n_1x_1 = n_2x_2 = \cdots = n_kx_k = 0 \tag{2}$$

or

$$n_1x_1 + n_2x_2 + \cdots + n_kx_k \neq 0 \tag{3}$$

In other words, no linear combination (3) *can be zero unless every summands is zero,* [10].

Theorem 3.7. *Let $K \subseteq \mathrm{C}$ be an infinite compact set, and take $\alpha \in (0, 1)$. Then $l^\alpha(K, A)$ is not amenable.*
Proof. Let $x_0 \in K$. We define

$$M_{x_0} := \{f \in l^\alpha(K, A) : \quad (\sigma of)(x_0) = 0 \quad \forall \sigma \in A^*\}$$

If $\sigma \in A^*$, then for each $f \in M_{x_0}^2$ we have

$$\frac{(\sigma o f)(x)}{d(x, x_0)^{2\alpha}} \longrightarrow 0 \quad as \quad d(x, x_0) \longrightarrow 0.$$

For $\beta \in (\alpha, 2\alpha)$, set $f_\beta(x) := \eta(d(x, x_0)^\beta)$, $x \in K$ where, the map $\eta : \mathrm{C} \to A$ defined by $\eta(\lambda) = \lambda.e$. Then $f_\beta \in M_{x_0}$ and $\{f_\beta + M_{x_0}^2 : \quad \beta \in (\alpha, 2\alpha)\}$ is a linearly independent set in $\frac{M_{x_0}}{M_{x_0}^2}$ because x_0 is non-isolated in K. Therefor $M_{x_0}^2$ has infinite codimension in M_{x_0} , and so $M_{x_0} \neq M_{x_0}^2$ then by [5] M_{x_0} has not a bounded approximate identity, and since M_{x_0} is closed ideal in $l^\alpha(K, A)$, then $l^\alpha(K, A)$ is not amenable. △

Theorem 3.8. *Let (K, d) be a compact metric space and A be a unital commutative Banach algebra. If $\frac{1}{2} < \alpha < 1$, then $l^\alpha(\mathrm{T}, A)$ is not weakly amenable, where T is unit circle in complex plane.*
Proof. By remark 7, we have $l^\alpha(\mathrm{T}, A) \cong l^\alpha(\mathrm{T})\check{\otimes}A$. Since by [5] $l^\alpha(\mathrm{T})$ is not weakly amenable, hence $l^\alpha(\mathrm{T}, A)$ is not weakly amenable. △

Corollary3.9. *Let A be a finite-dimensional weakly amenable Banach algebra. If $0 < \alpha < \frac{1}{2}$, then $l^\alpha(K, A)$ is weakly amenable.*
Proof. By [11], $l^\alpha(K)\hat{\otimes}A$ is weakly amenable. Now by [11], we have $l^\alpha(K)\hat{\otimes}A \cong l^\alpha(K)\check{\otimes}A$ and this implies that $l^\alpha(K)\check{\otimes}A$ is weakly amenable and so $l^\alpha(K, A)$ is weakly amenable. △

References

[1] Alimohammadi, D. and Ebadian, A. *Headberg's theorem in real Lipschitz algebras*, Indian J. Pure Appl. Math, 32, (2001), 1479-1493.

[2] Bade, W. G., Curtis, P. C. and Dales, H. G., *Amenability and weak amenability for Berurling and Lipschitz algebras*, Proc. London. Math. Soc. (3), 55 (1987), 359-377, .

[3] Cao, H. X. and Xu, Z. B., *Some properties of Lipschitz-α operators*, Acta Mathematica Sinica, English Series, 45 (2), (2002), 279-286 .

[4] Cao, H. X., Zhang, J. H. and Xu, Z. B., *Characterizations and extentions of Lipschitz-α operators*, Acta Mathematica Sinica, English Series, 22 (3), (2006), 671-678 .

[5] Dales, H. G., *Banach algebras and Automatic Continuty*, Clarendon press. oxford, 2000.

[6] Ebadian, A., *Prime ideals in Lipschitz algebras of finite differentable function*, Honam Math. J., 22 (2000), 21-30, .

[7] Honary, T. G, Mahyar, H., *Approximation in Lipschitz algebras*, Quest. Math. 23, (2000), 13-19 .

[8] Johnson, B. E., *Cohomology in Banach algebras*, Men. Amer. Soc, 127 (1972).

[9] Johnson, B. E., *Lipschitz spaces*, Pacfic J. Math, 51, (1975), 177-186.

[10] Rudin, W. *Fourier analysis on groups*, Wiley, 1990.

[11] Runde, Volker, *Lectures on amenability*, Springer, 2001.

[12] Sherbert, D. R., *Banach algebras of Lipschitz functions*, Pacfic J. Math, 13, (1963), 1387-1399.

[13] Sherbert, D. R., *The structure of ideals and point derivations in Banach algebras of Lipschitz functions*, Trans. Amer. Math. Soc., 111, (1964), 240-272.

[14] Weaver, N., *Subalgebras of little Lipschitz algebras*, Pacfic J. Math., 173, (1996), 283-293.

[15] Weaver, N., *Lipschitz algebras*, World Scientific Publishing Co., Inc., River Edge, NJ, 1999.

Authors:

A.A. Shokri
Faculty Of Basic Science, Science And Research Brunch,
Islamic Azad University (IAU), Tehran, Iran.
email:*a-shokri@iau-ahar.ac.ir*

A. Ebadian
Department of mathematics, Faculty of science,
Urmia University, Urmia, Iran.
email:*a.ebadian@urmia.ac.ir*

A.R. Medghalchi
Department of Mathematics,Tarbiat Moallem
University, 599 Talegani Avenue, Tehran, Iran.
email:*a-medghalchi@saba.tmu.ac.ir*